DOCUMENT RESUME

ED 069 535 SE 015 346

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TITLE	Similarity and Congruence.
INSTITUTION	Oakland County Schools, Pontiac, Mich.
SPONS AGENCY	Bureau of Elementary and Secondary Education (DHEW/OE), Washington, D.C.
PUB DATE	Mar 71
GRANT	OEG-68-05635-0
NOTE	53p.; Revised Edition
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	Classification; *Congruence; Curriculum; *Geometric Concepts; Instruction; *Instructional Materials; Low Ability Students; Mathematics Education; Objectives; Ratios (Mathematics); *Secondary School Mathematics; Units of Study (Subject Fields); Worksheets
IDENTIFIERS	ESEA Title III

ABSTRACT

This instructional unit is an introduction to the common properties of similarity and congruence. Manipulation of objects leads to a recognition of these properties. The ASA, SAS, and SSS theorems are not mentioned. Limited use is made in the application of the properties of size and shape preserved by similarity or congruence. A teacher's guide is available. Related documents are SE 015 334 - SE 015 345 and SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

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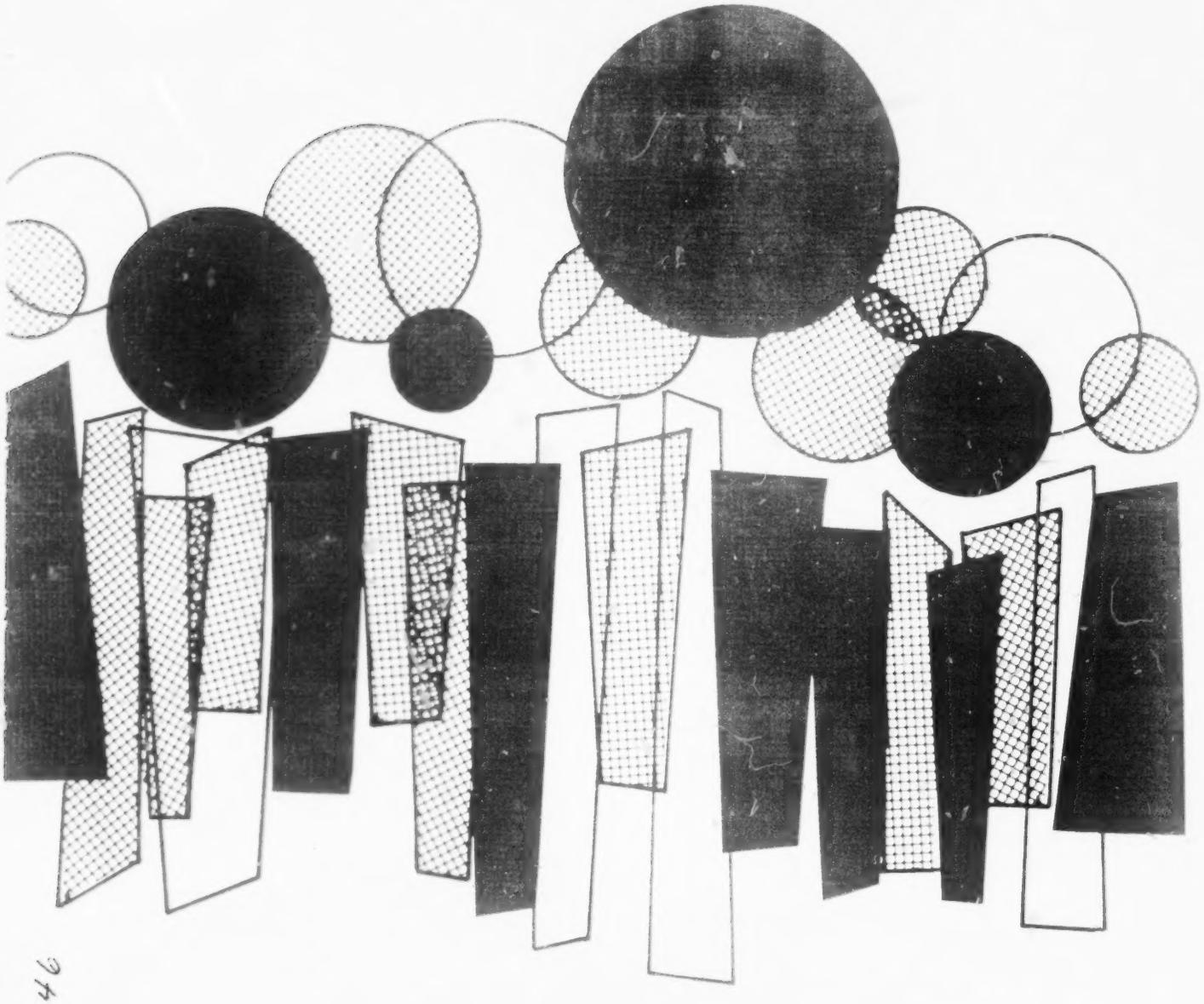
SIMILARITY AND CONGRUENCE



SE 015 346

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SIMILARITY AND CONGRUENCE

OAKLAND COUNTY MATHEMATICS PROJECT

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REVISED EDITION — MARCH, 1971

This unit was prepared under the auspices of U.S.O.E. Grant 68–05635–0, Title III, E.S.E.A.

PREFACE

Two figures or objects that are the same shape have more in common than their shapes. Can you think of some of these common properties? Similarity and Congruence is a study of figures and objects whose sizes and/or shapes are exactly the same.

Architects use some of the principles of similarity to draw blueprints, and surveyors use the same principles when they draw maps. Locksmiths use the notion of congruence when they make a copy of a key, and automobile manufacturers use the same notion of congruence to mass-produce parts for cars.

This booklet is written to help you learn some of the common properties of similarity and congruence and to let you use these properties. When you are finished with this booklet, you will know several of these properties and some of their applications.

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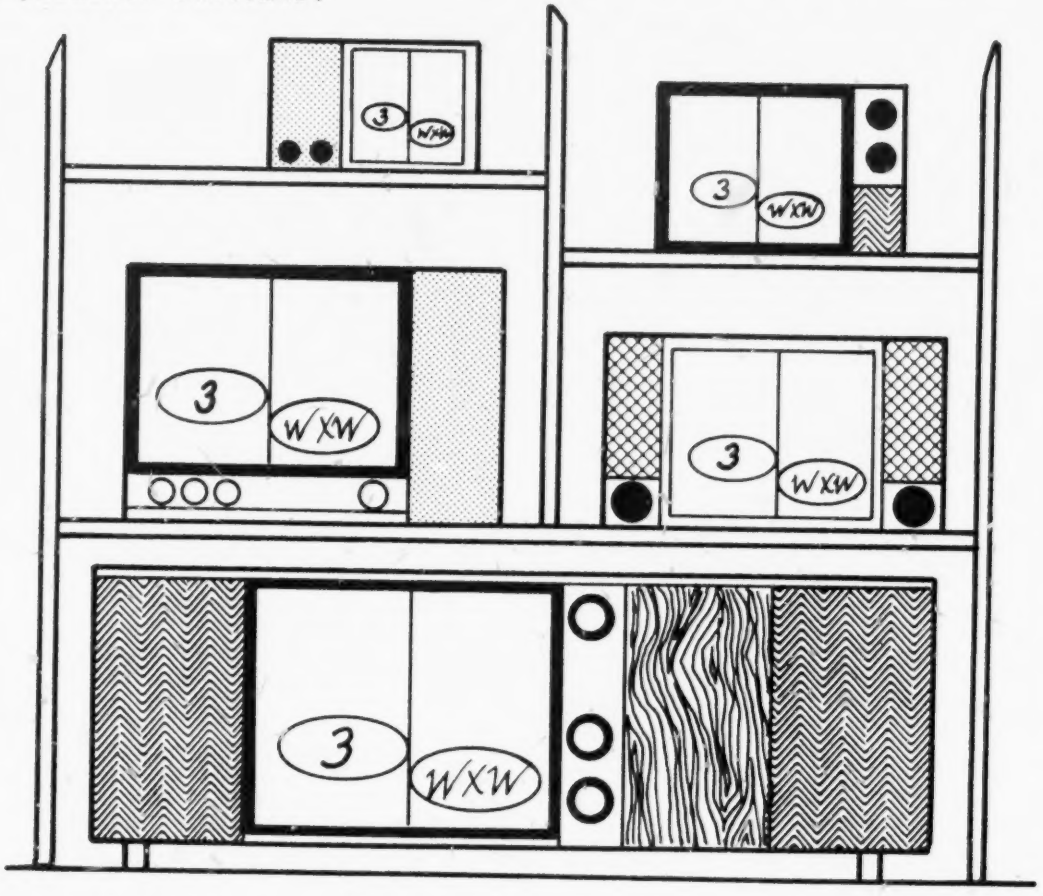
LESSON 1

SIZE AND SHAPE

The manager of Kelly's Appliance Store frequently tunes all the television sets in his store to the same station. This never fails to attract the attention of those who walk by and usually provides his salesman with an opportunity to talk to a possible customer.



The manager and the salesman disagree on whether the pictures are actually the same. The salesman claims the pictures are exactly the same and the manager says they are different. Who do you agree with? Can you name some reasons why each man feels he is right?

THE PICTURES ON THE TELEVISIONS ARE EXACTLY THE SAME SHAPE.

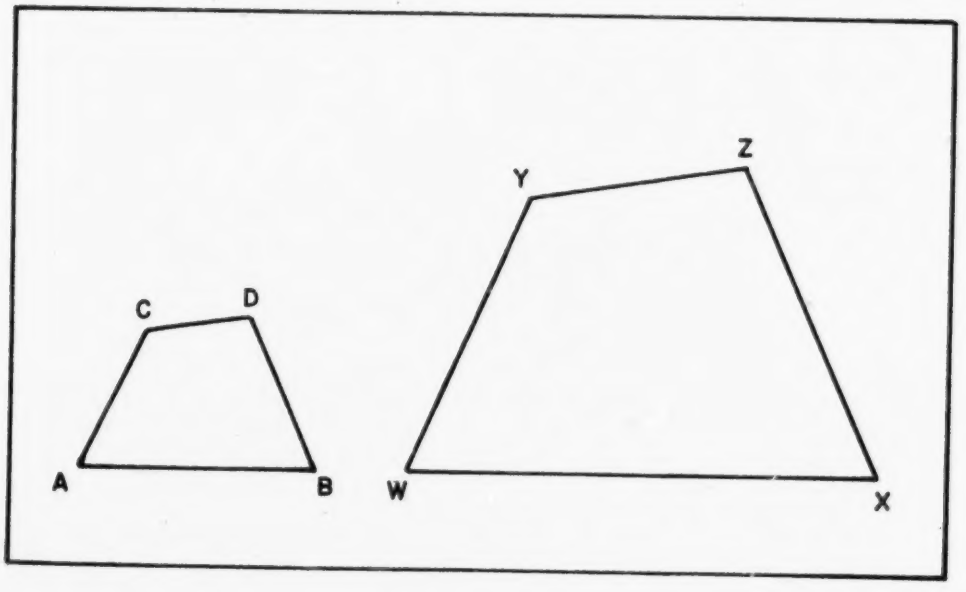
CLASS DISCUSSION

1. Do you agree with the statement above? What does it mean to say two pictures or objects have the same shape?

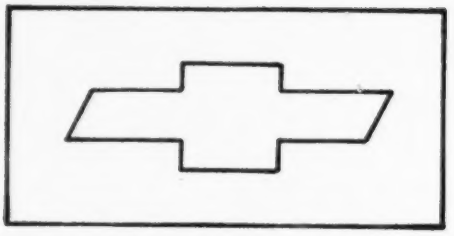
2. Photographs can be enlarged, but they remain the same shape. Can you name some other objects which vary in size but maintain the same shape?

3. Do all maps of Michigan have the same shape? How can you be sure the maps give an accurate picture of the actual shape of the state? Do you have any idea how maps are made?

GEOMETRIC FIGURES WHICH ARE EXACTLY THE SAME SHAPE ARE CALLED SIMILAR FIGURES.

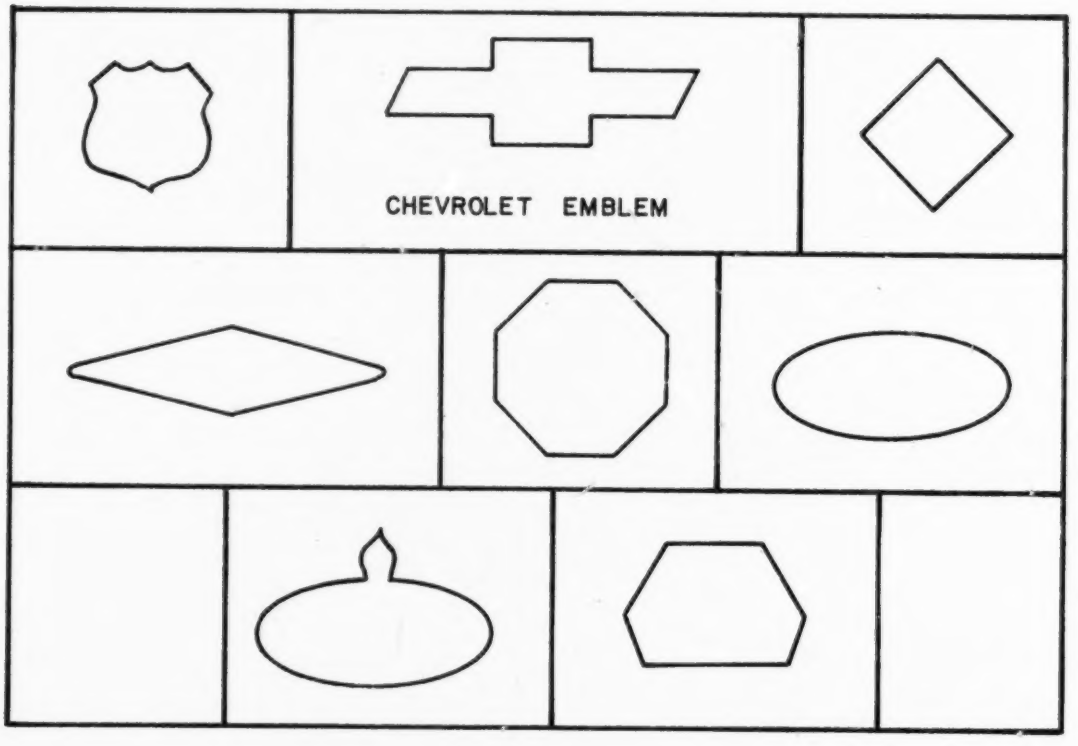


HAVE YOU SEEN THIS FIGURE BEFORE?



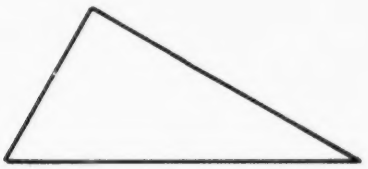
General Motors uses this figure as an emblem to identify and promote Chevrolet automobiles and trucks. You've seen the emblem displayed in several places including newspapers, television and billboards. Is the emblem on a billboard the same size as one in a newspaper? Are they both the same shape? The Chevrolet emblem may be very large or very small, but it is always the same shape. Geometrically speaking, the emblems are similar figures.

Can you identify a familiar sign whose shape is similar to the figures below?

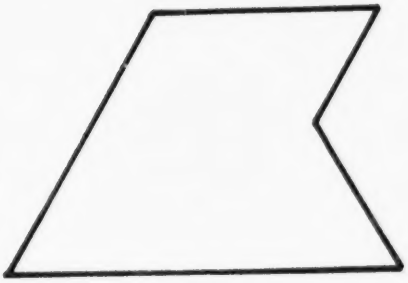


Tear out pages 4a and 4b and cut along the dotted lines. Are figures 1a and 1b similar? _____ Try placing one over the other and holding them to the light. Move them around, flip one over, compare the angles; continue until you can see the similarity or are satisfied that no matter how you line them up, the figures are not similar. Use this method to compare 2a and 2b, 3a and 3b etc. Use the chart to indicate your decision.

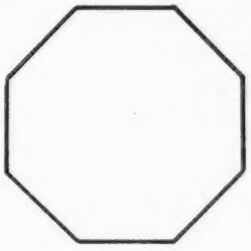
Figures	Similar	Not Similar
1a and 1b		
2a and 2b		
3a and 3b		
4a and 4b		
5a and 5b		
6a and 6b		
7a and 7b		
8a and 8b		
9a and 9b		
10a and 10b		



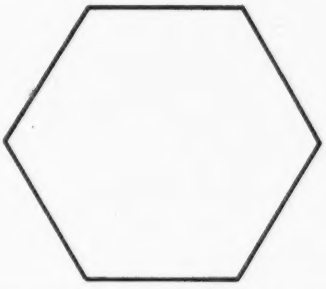
1a



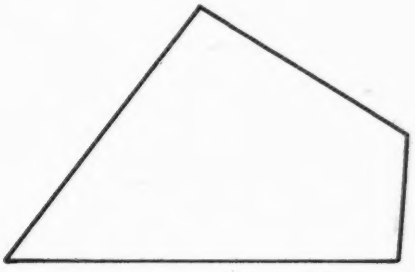
6a



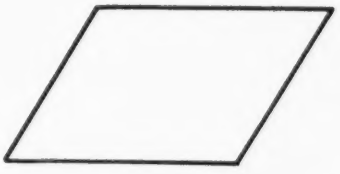
2a



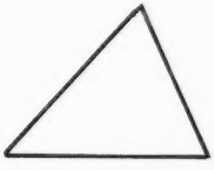
7a



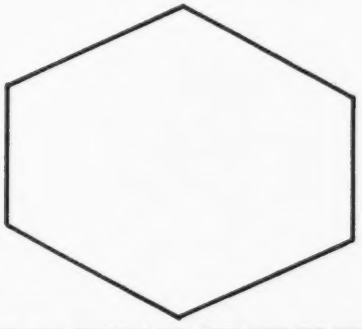
3a



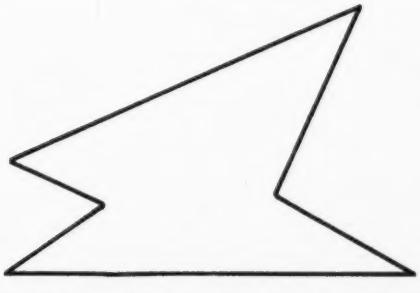
8a



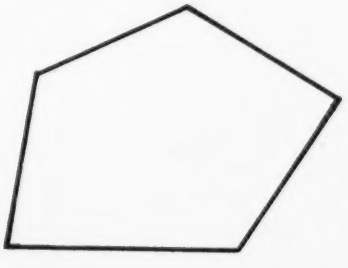
4a



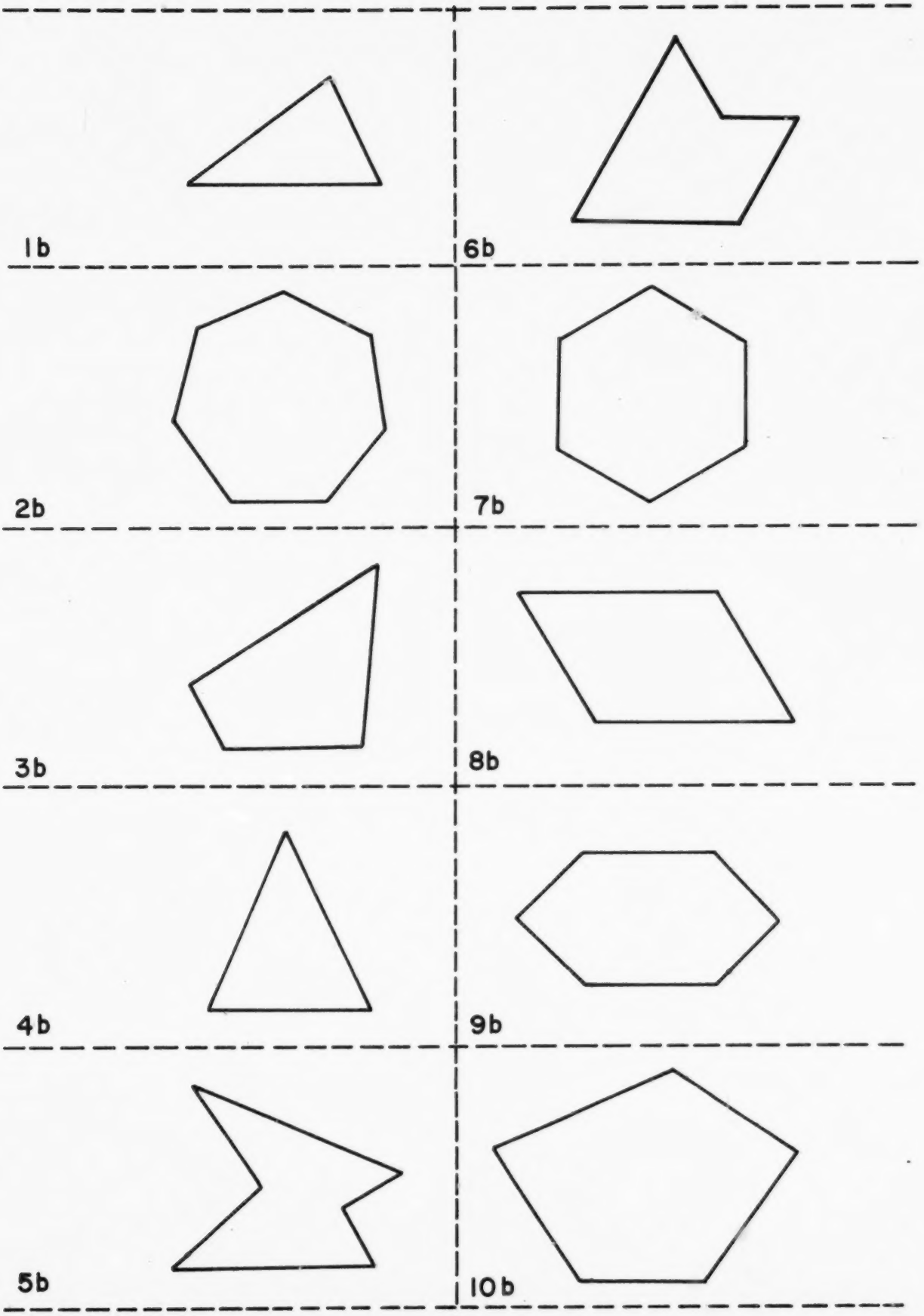
9a



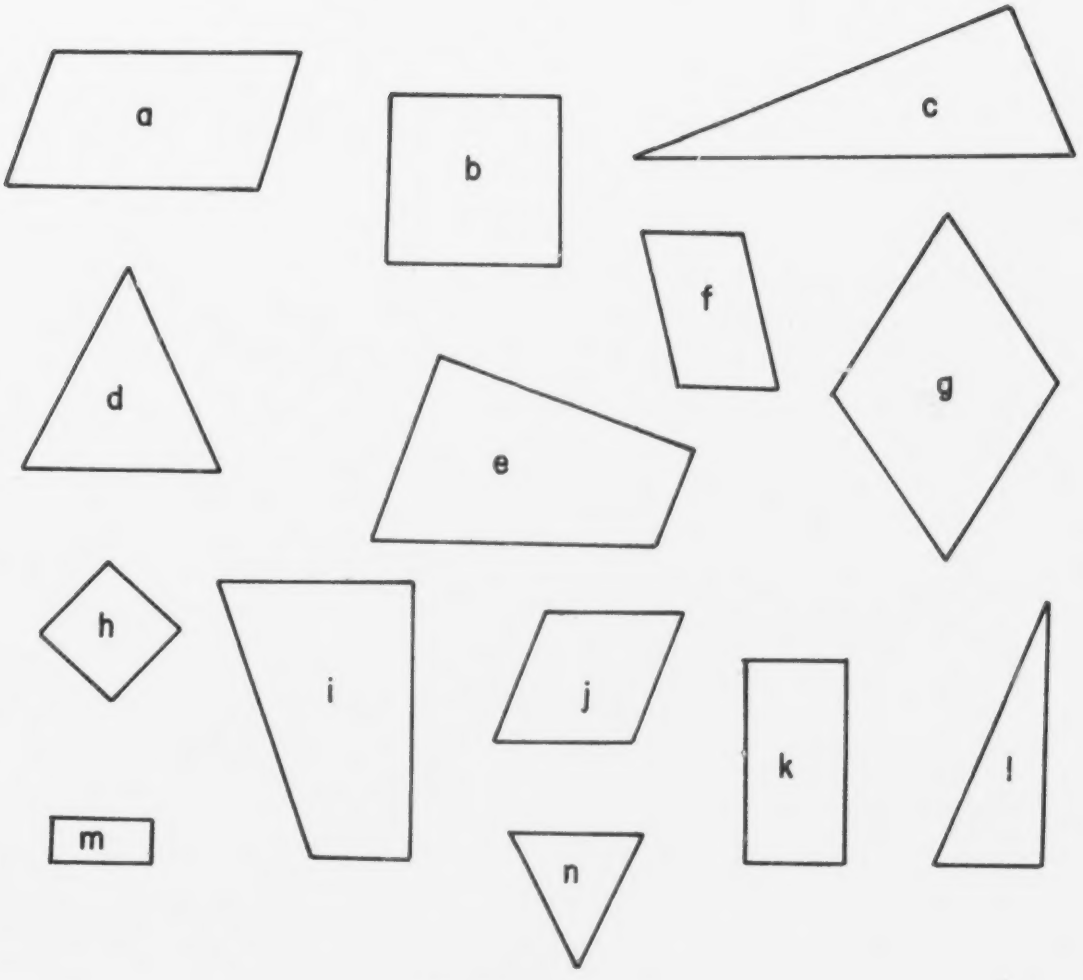
5a



10a


1b
6b
2b
7b
3b
8b
4b
9b
5b
10b

FIND THE SIMILAR FIGURES



_____ is similar to _____

_____ is similar to _____

_____ is similar to _____

_____ is similar to _____

_____ is similar to _____

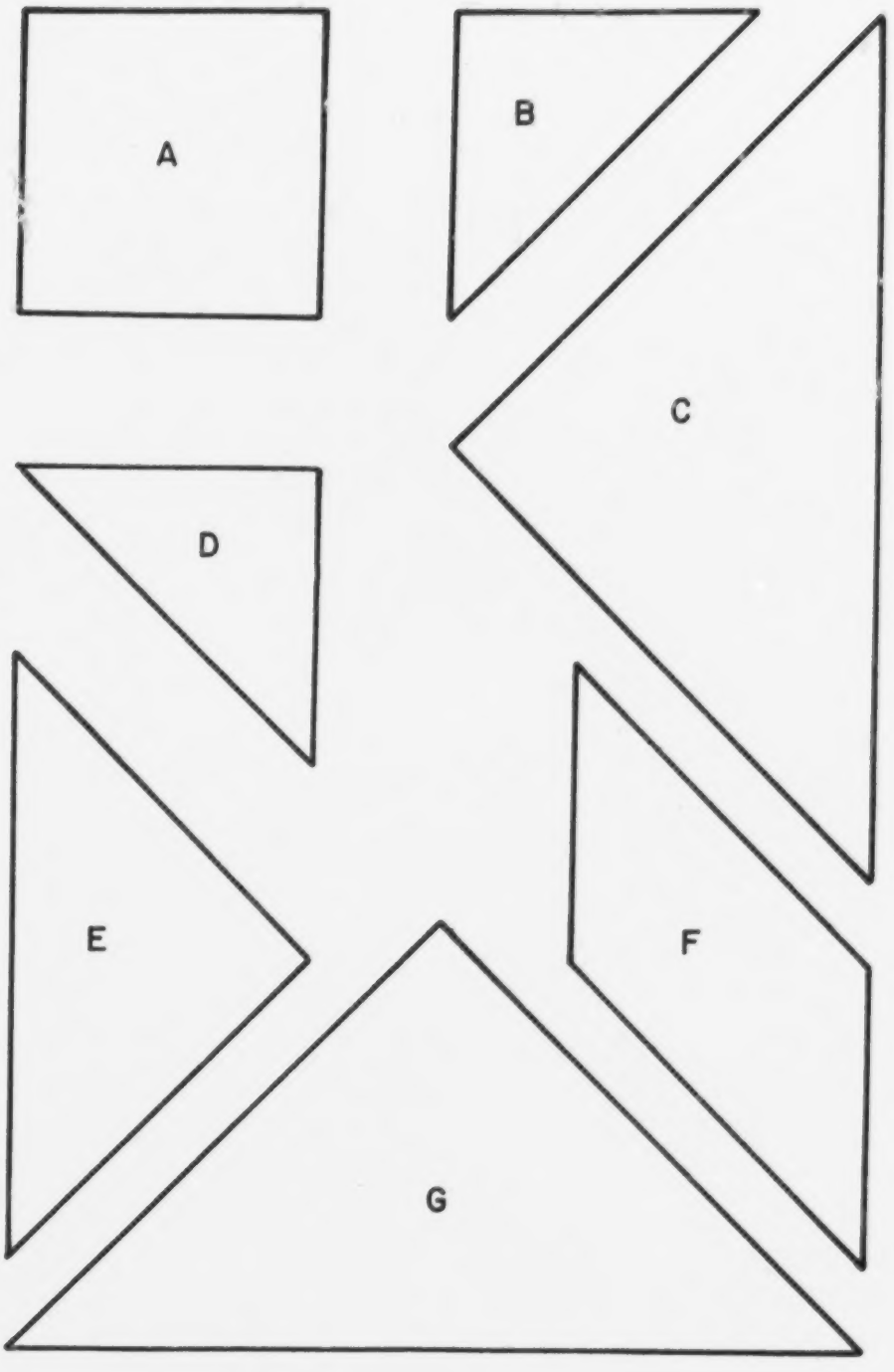
_____ is similar to _____

_____ is similar to _____

Cut out insert A and carefully cut out each figure. Which figures are similar to each other? _____ Show how figures B and D can be combined to form a figure the same size and shape as figure A, E or F. Name some combinations which will form a figure the same size and shape as figure C.

Are the statements true or false?

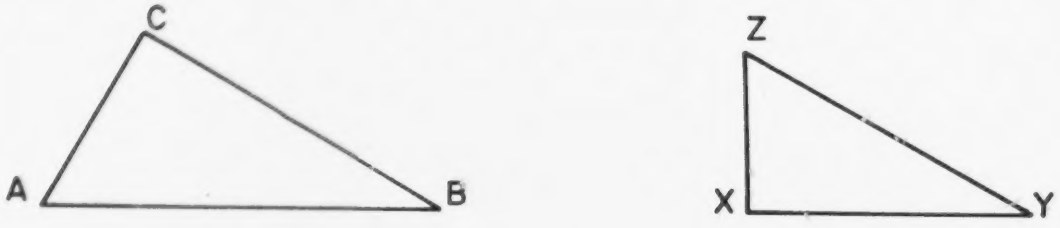
1. Figures A, B and D will form a figure similar to figure F.
2. Figures B, D and E will form a figure similar to figure A.
3. Figures A and B will form a figure similar to figure E.
4. Figures C and G will form a figure similar to figure F.
5. Figures B, D and F will form a figure similar to figure A.
6. Figures D, E and F will form a figure similar to figure A.
7. Figures G and C will form a figure similar to figure E.
8. Figures A, B, D, E and F will form a figure similar to figure C.
9. Figures B, C, D and E will form a figure similar to figure F.
10. Figures D, B, E and G will form a figure similar to figure F.
11. All the figures will form a figure similar to figure A.
12. All the figures will form a figure similar to figure C.



INSERT A

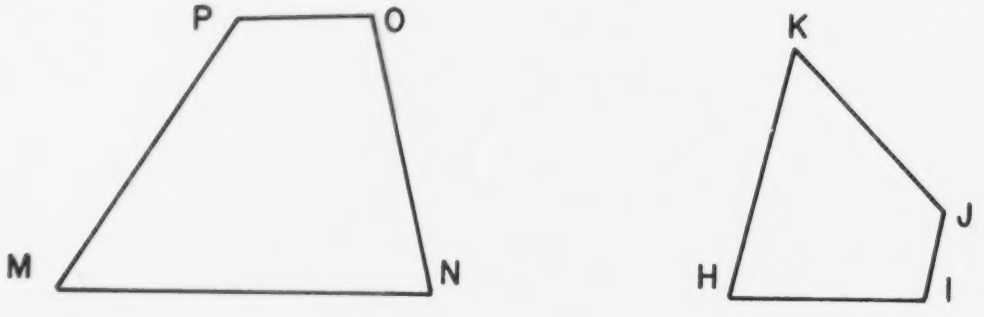
COMPARE THE ANGLES

Triangle ABC is similar to triangle ZYX ($\triangle ABC \sim \triangle ZYX$).



Angle C is the largest angle of triangle ABC and angle X is the largest angle of triangle ZYX. We say that angle C corresponds to angle X. Angle B is the smallest angle of triangle ABC. Which angle of triangle ZYX corresponds to angle B?_____ Which angle of triangle ZYX corresponds to angle A?_____

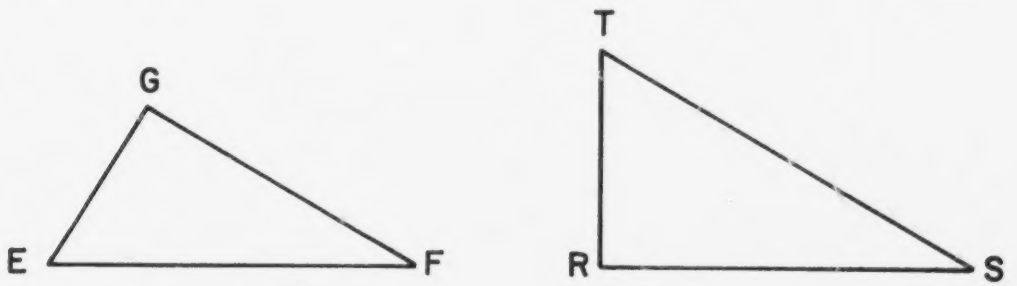
Figure MNOP is similar to Figure KHIJ (fig. MNOP~fig. KHIJ).



Angle M corresponds to angle K. Use a compass, protractor or tracing paper to compare their sizes. Is angle M less than, greater than, or the same size as angle K? _____
Angle N corresponds to angle H. How do their sizes compare? _____Angle O corresponds to angle I. How do their sizes compare? Is angle P the same size as angle J? _____
Compare the sizes of the corresponding angles of the triangles at the top of this page.

THE CORRESPONDING ANGLES OF SIMILAR FIGURES ARE THE SAME SIZE.

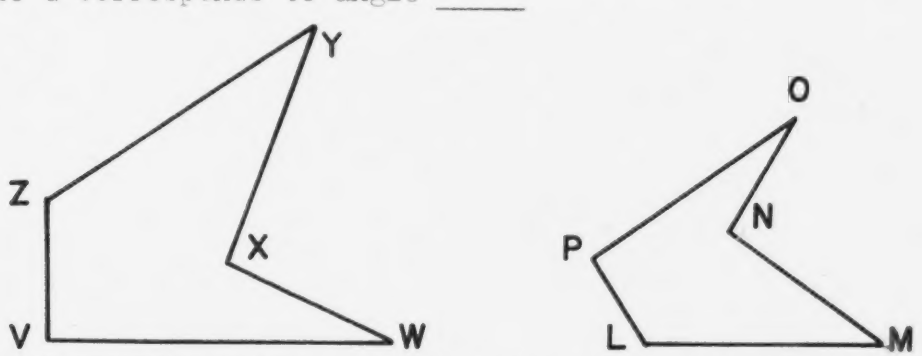
Triangle EFG is similar to triangle TSR ($\triangle EFG \sim \triangle TSR$)



Which angle corresponds to angle E? (Find the angle which is the same size as angle E.) _____

Which angle corresponds to angle F? _____

Angle G corresponds to angle _____

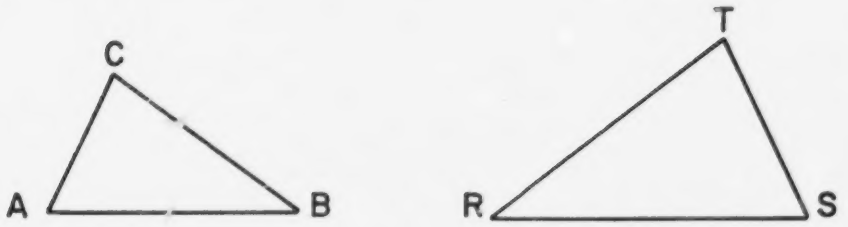


Find the corresponding angles of the similar figures above.

Angle V corresponds to angle _____
Angle W corresponds to angle _____
Angle X corresponds to angle _____
Angle Y corresponds to angle _____
Angle Z corresponds to angle _____

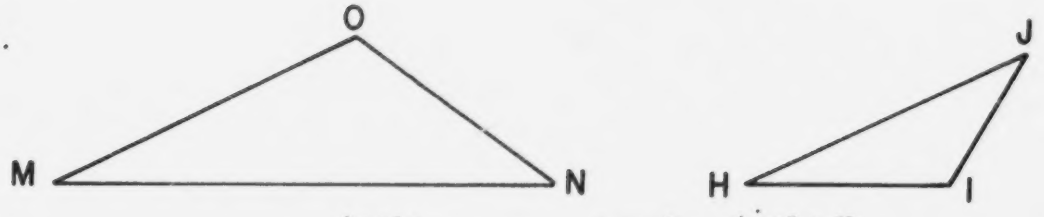
Find the corresponding angles of the similar figures.

1.



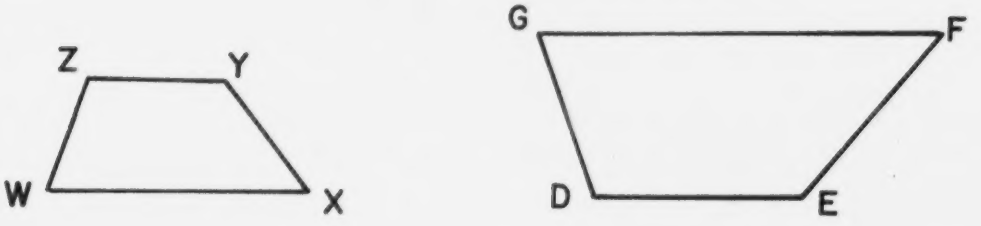
Angle A ⟷ Angle _____
Angle B ⟷ Angle _____.
Angle C ⟷ Angle _____

2.



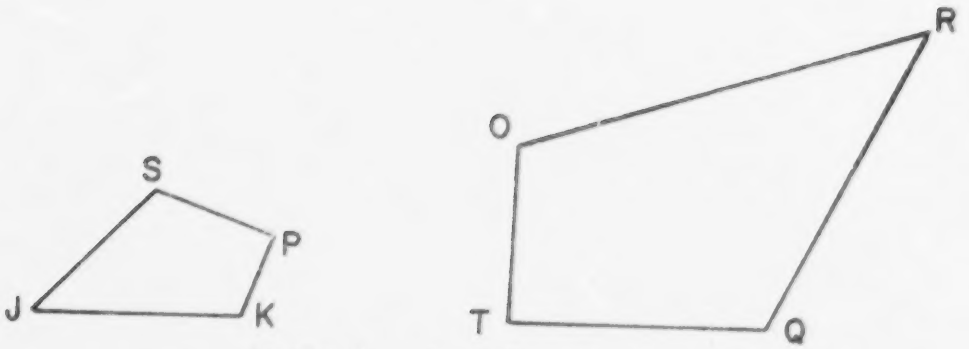
Angle _____ ⟷ Angle H
Angle _____ ⟷ Angle I
Angle _____ ⟷ Angle J

3.



Angle W ⟷ Angle _____
Angle X ⟷ Angle _____
Angle Y ⟷ Angle _____
Angle Z ⟷ Angle _____

4.



Angle S ⟷ Angle _____

Angle J ⟷ Angle _____

Angle K ⟷ Angle _____

Angle P ⟷ Angle _____

5.

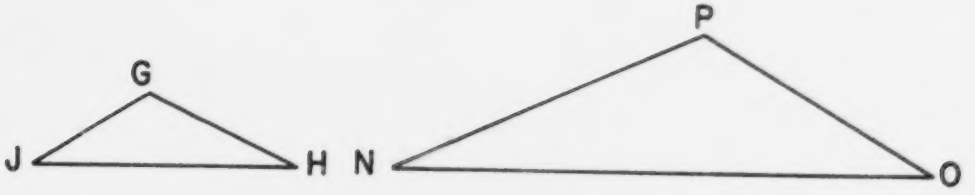
Angle _____ ⟷ Angle B

Angle _____ ⟷ Angle Q

Angle _____ ⟷ Angle D

Angle _____ ⟷ Angle L

6.



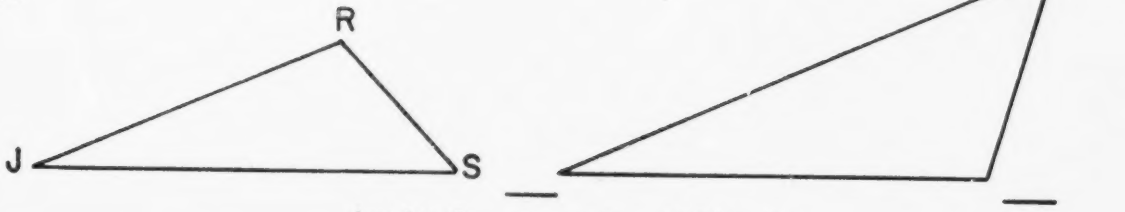
Angle H ⟷ Angle _____

Angle G ⟷ Angle _____

Angle J ⟷ Angle _____

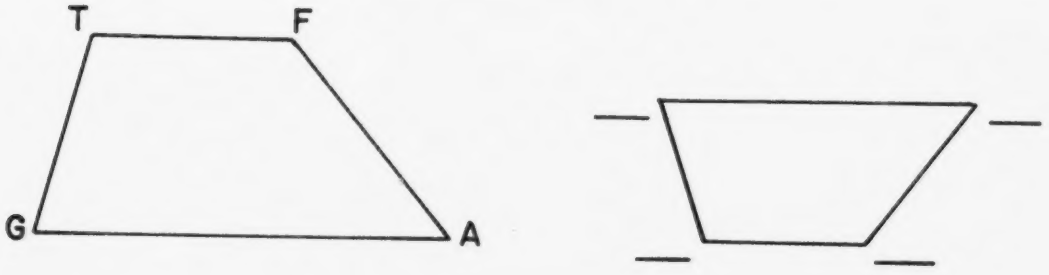
The corresponding angles are given. Use this information to correctly label the vertices of the second figure.

7.



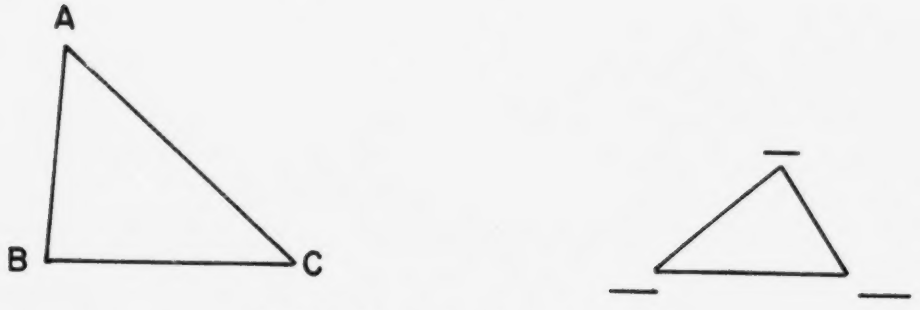
Angle R ⟷ Angle D
Angle J ⟷ Angle F
Angle S ⟷ Angle E

8.



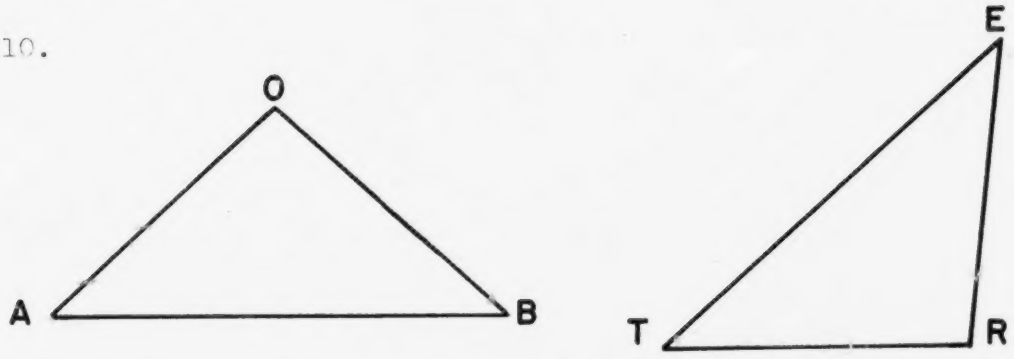
Angle G ⟷ Angle S
Angle A ⟷ Angle N
Angle F ⟷ Angle K
Angle T ⟷ Angle D

9.



Angle A ⟷ Angle Z
Angle B ⟷ Angle Y
Angle C ⟷ Angle X

10.



The similar triangles above have two correspondences which pair up angles of the same size. One is given. Can you find the other?

Angle A ⟷ Angle T
Angle B ⟷ Angle E
Angle O ⟷ Angle R

Angle A ⟷ Angle _____
Angle B ⟷ Angle _____
Angle O ⟷ Angle _____

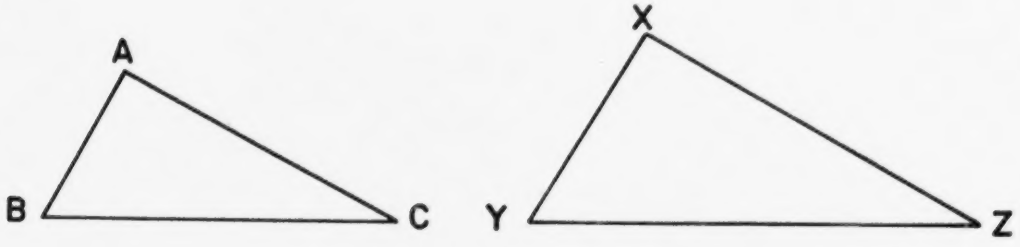
11. Describe a pair of triangles having three correspondences which pair up angles of the same size.

✓ **POINT**

1. Can you give a brief definition of similar figures?
2. What relationship do corresponding angles of similar figures have?
3. Explain your procedure for determining the corresponding angles of two similar triangles.

SIMILARITY CORRESPONDENCE

Triangle ABC is similar to triangle XYZ ($\triangle ABC \sim \triangle XYZ$).



Since the triangles are similar, the corresponding angles are the same size. The corresponding angles are:

$$\angle A \longleftrightarrow \angle X$$
$$\angle B \longleftrightarrow \angle Y$$
$$\angle C \longleftrightarrow \angle Z$$

A shorter way to represent the corresponding angles of the triangles is

$$\triangle ABC \longleftrightarrow \triangle XYZ$$

Notice how the corresponding angles are represented in this expression

$$\triangle A\ B\ C \longleftrightarrow \triangle X\ Y\ Z$$

We have made a correspondence between the similar triangles which also indicates the corresponding angles.

A correspondence between two similar figures which names the corresponding angles is called a SIMILARITY CORRESPONDENCE.

When we say triangle ABC is similar to triangle XYZ or write $\triangle ABC \sim \triangle XYZ$, we understand this to mean that:

1. Triangle ABC corresponds to triangle XYZ ($\triangle ABC \leftrightarrow \triangle XYZ$)
2. The corresponding angles are the same size.

EXERCISES

1. Name the corresponding angles of the similar triangles named below.

a) $\triangle RST \sim \triangle DEF$

b) $\triangle UVW \sim \triangle JHI$

c) $\triangle ACD \sim \triangle WYX$

d) $\triangle MNO \sim \triangle ONP$

2. The corresponding angles are given. Name the similar triangles.

a) $\angle A \longleftrightarrow \angle O$
$\angle B \longleftrightarrow \angle M$
$\angle C \longleftrightarrow \angle N$

b) $\angle W \longleftrightarrow \angle H$
$\angle X \longleftrightarrow \angle K$
$\angle Y \longleftrightarrow \angle J$

3. The corresponding angles of a pair of similar triangles are:

$\angle B \longleftrightarrow \angle O$
$\angle C \longleftrightarrow \angle N$
$\angle D \longleftrightarrow \angle P$

Which of the following names a similarity correspondence between these triangles? (More than one may be correct.)

a) $\triangle BCD \longleftrightarrow \triangle ONP$

b) $\triangle DCB \longleftrightarrow \triangle PNO$

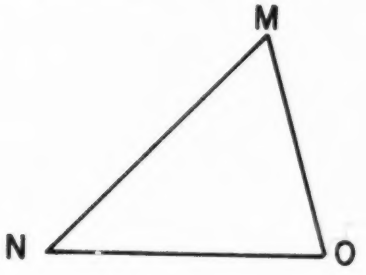
c) $\triangle CDB \longleftrightarrow \triangle NOP$

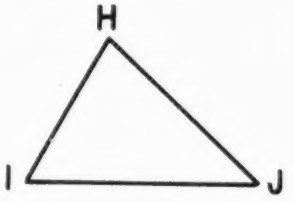
d) $\triangle DBC \longleftrightarrow \triangle PON$

e) $\triangle BDC \longleftrightarrow \triangle ONP$

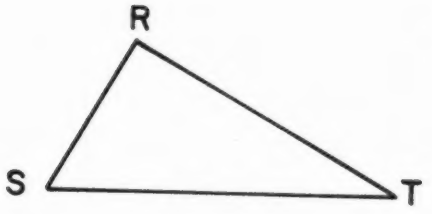
4. In each part below, the figures are similar. Name a similarity correspondence for each pair of figures.

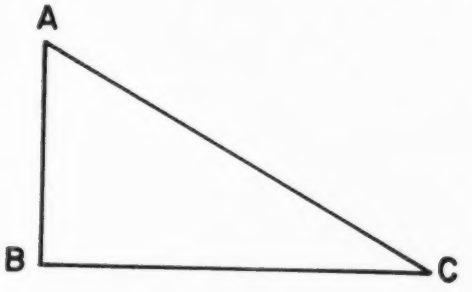
a. $\triangle$ MNO $\sim$ _____



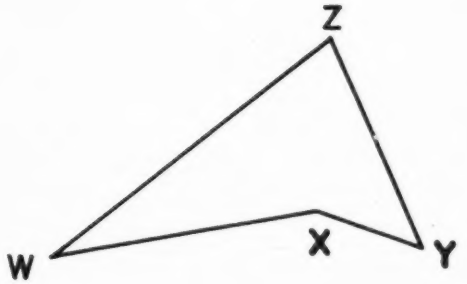


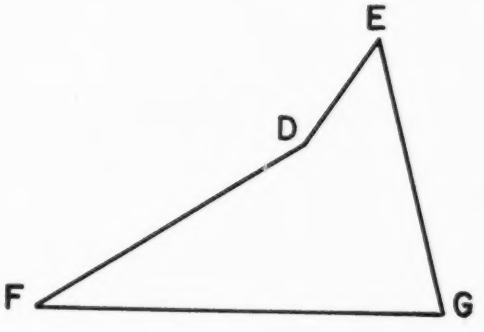
b. $\triangle$ RST $\sim$ _____



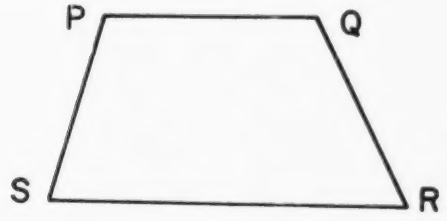


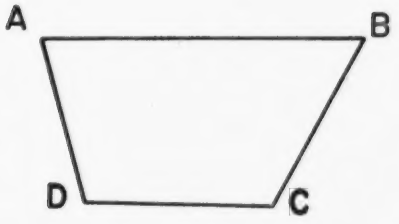
c. fig. WXYZ $\sim$ _____



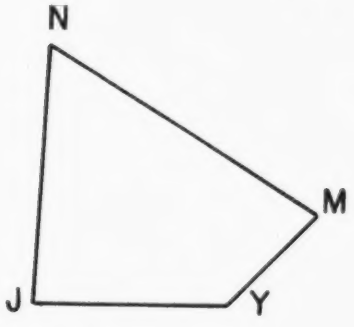


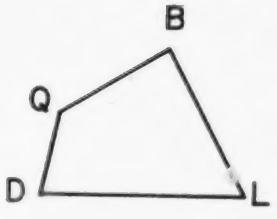
d. fig. ABCD $\sim$ _____





e. fig. NJYM $\sim$ _____

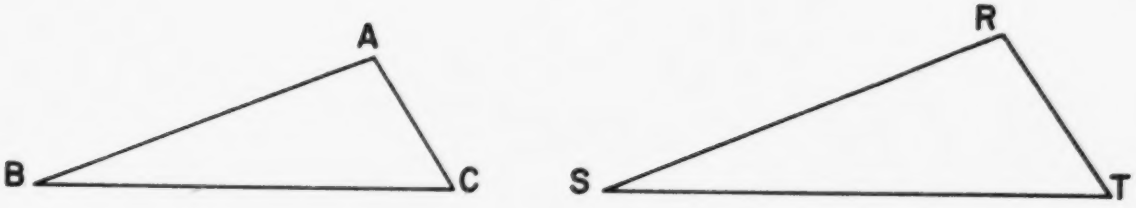




f. ΔNOP $\sim$ _____

CORRESPONDING SIDES

Besides naming the corresponding angles, a similarity correspondence between two figures also names the corresponding sides of the figures.



Triangle ABC is similar to triangle RST ($\triangle ABC \sim \triangle RST$)

A B C ~ R S T — Side AB corresponds to side RS ($\overline{AB} \longleftrightarrow \overline{RS}$)

A B C ~ R S T — Side BC corresponds to side ST ($\overline{BC} \longleftrightarrow \overline{ST}$)

A B C ~ R S T — Side AC corresponds to side RT ($\overline{AC} \longleftrightarrow \overline{RT}$)

DISCUSSION QUESTIONS

1. Given any similarity correspondence, make a rule for naming corresponding sides of the similar figures.
2. The corresponding angles of similar figures are the same size. What relationship do corresponding sides have?
3. Given two similar figures, describe a procedure for finding a similarity correspondence.
4. If the corresponding angles of a pair of similar figures are known, is it possible to determine the corresponding sides? Explain your answer.

EXERCISES

1) Name the corresponding sides of the similar triangles below.

a. $\triangle RST \sim \triangle DEF$ c. $\triangle ACD \sim \triangle WYZ$

b. $\triangle UVW \sim \triangle JHI$ d. $\triangle MNO \sim \triangle ONP$

2) The corresponding sides are given. Name the similar figures.

a) $\overline{AB} \leftrightarrow \overline{OM}$
$\overline{BC} \leftrightarrow \overline{MN}$
$\overline{AC} \leftrightarrow \overline{ON}$

b) $\overline{WX} \leftrightarrow \overline{HK}$
$\overline{XY} \leftrightarrow \overline{KJ}$
$\overline{YZ} \leftrightarrow \overline{JI}$
$\overline{WZ} \leftrightarrow \overline{HI}$

3) The corresponding angles of a pair of similar triangles are

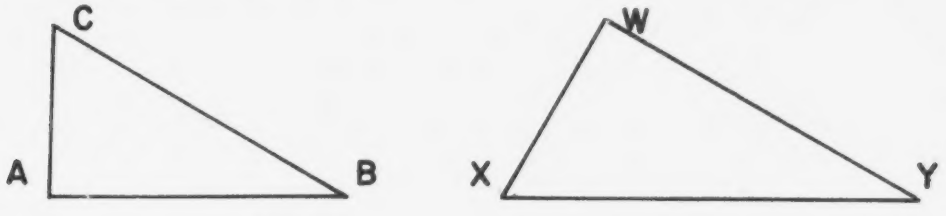
$\angle B \leftrightarrow \angle N$
$\angle A \leftrightarrow \angle T$
$\angle D \leftrightarrow \angle X$

a) Name the similar triangles.

b) Name the corresponding sides.

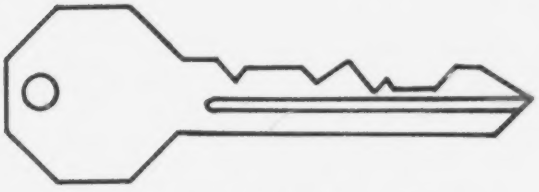
4) Name a similarity correspondence for the pairs of similar figures and complete the tables naming the corresponding sides.

a. $\triangle ABC \sim$ _____



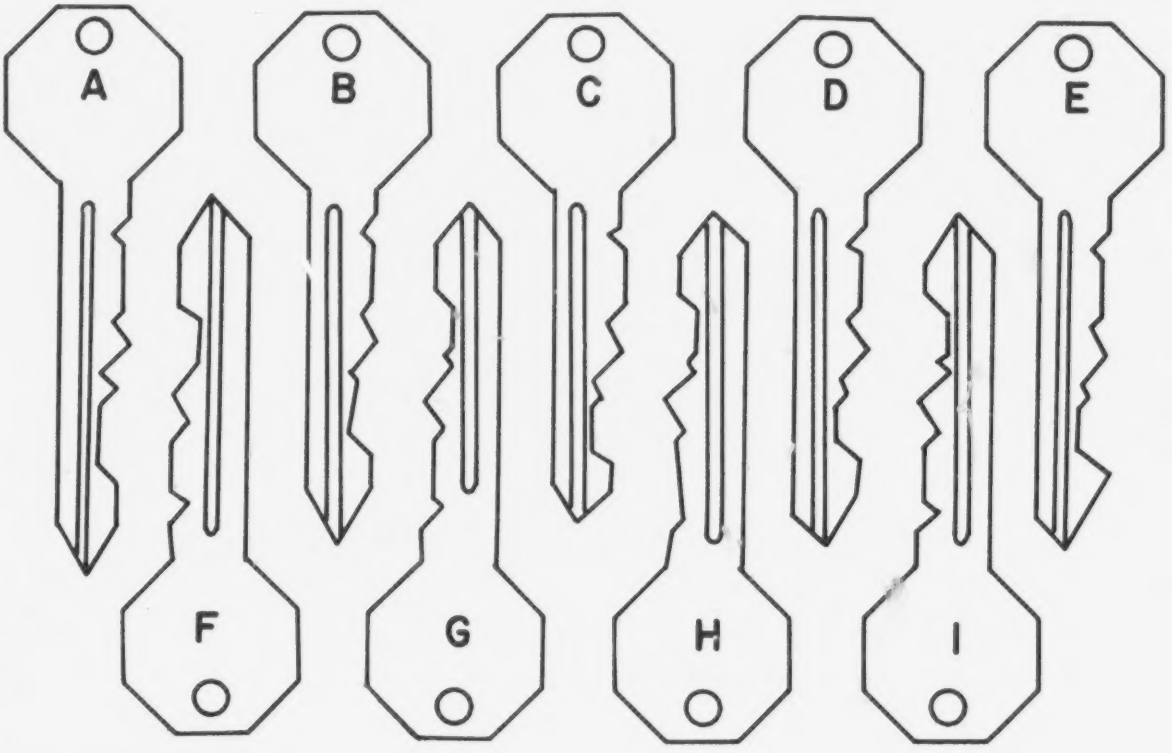
$\overline{AB} \leftrightarrow$ _____
$\overline{BC} \leftrightarrow$ _____
$\overline{CA} \leftrightarrow$ _____

A PERFECT MATCH



One of the keys below will unlock the same door as the key above.

Can you find it? Place a circle around your suggestion.

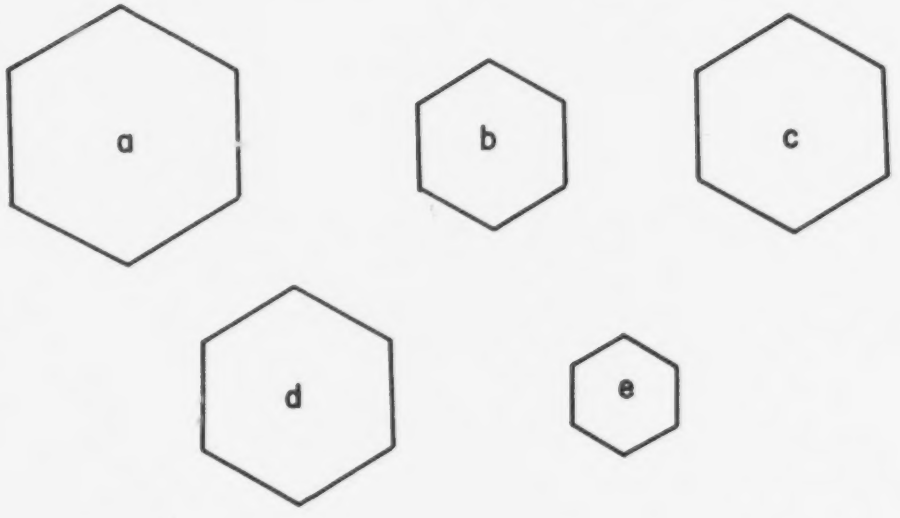


DISCUSSION QUESTIONS

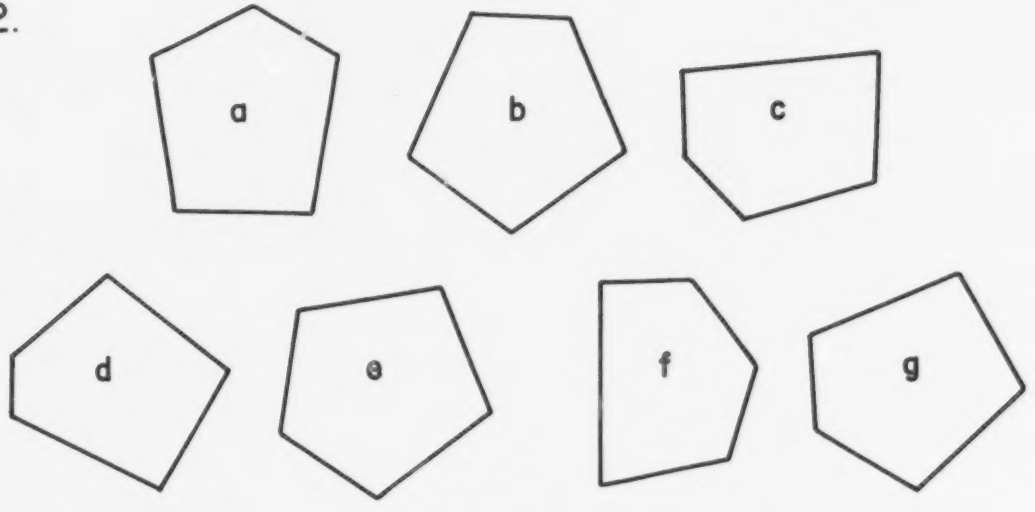
1. Why do you think the key you selected will unlock the door?
2. Can you think of a way to prove your selection is correct?
3. Explain your reasons for "ruling out" each of the other keys.

In each set of figures, find the pair or pairs which are exactly the same shape and size.

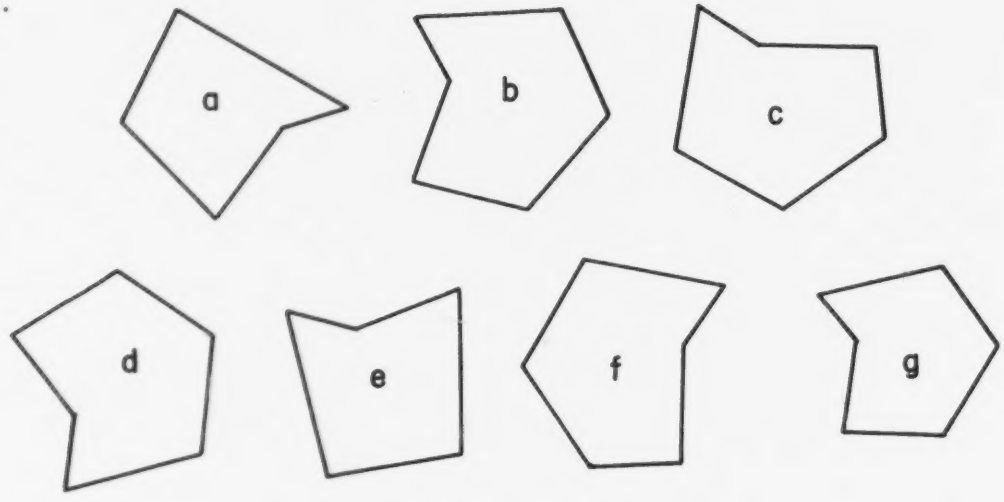
1.



2.

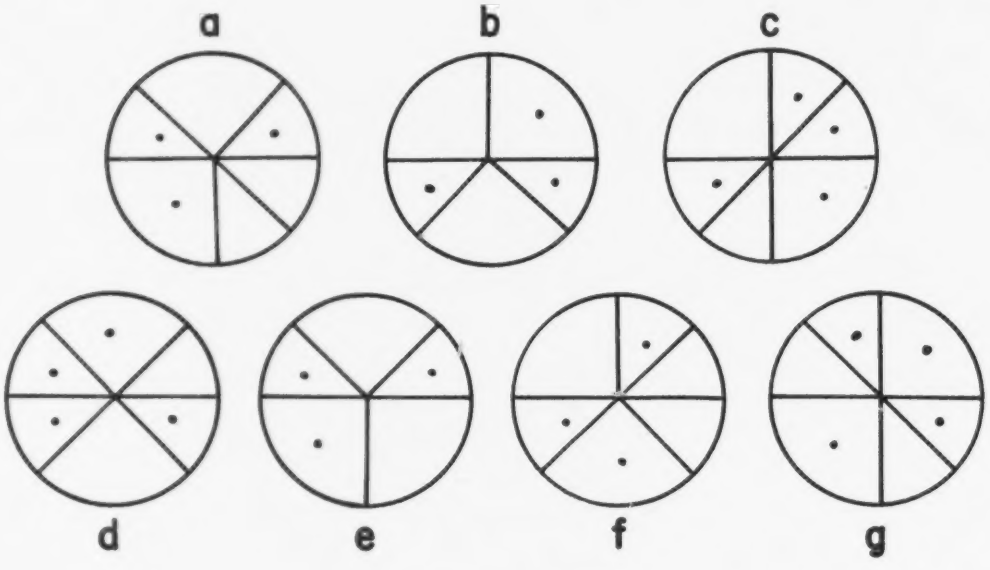


3.



If two figures are placed one over the other, and all their corresponding parts agree exactly, we say the figures coincide.

In the following sets of points, find the pairs which could be made to coincide.



CONGRUENCE

GEOMETRIC FIGURES WHICH ARE EXACTLY THE SAME SHAPE AND SIZE ARE CALLED CONGRUENT FIGURES.

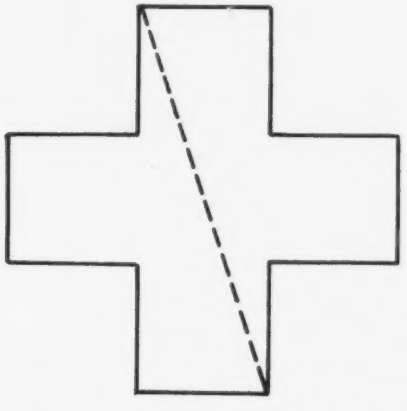
Which of the following figures would always be congruent?

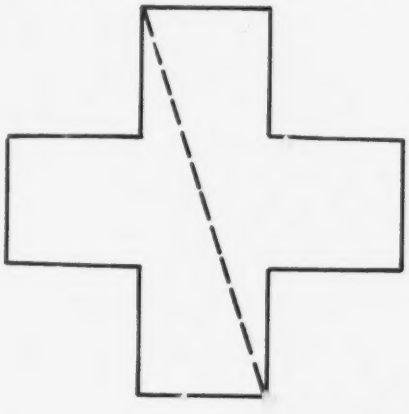
a. squares
b. right angles
c. rectangles
d. similar figures
e. acute angles
f. similar figures of the same size
g. right triangles with the same area
h. circles with the same radius
i. squares with the same area
j. rectangles with the same area

What applications would congruence have in the following list of items?

a. gaskets
b. coins
c. a rubber stamp
d. light bulbs
e. sports
f. an assembly line
g. maps
h. footprints

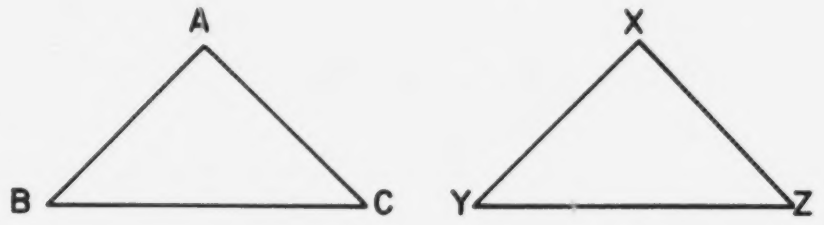
Make cutouts of the congruent figures below. Cut along the dotted lines. Are the four figures which result congruent? Use these four figures to form a square.





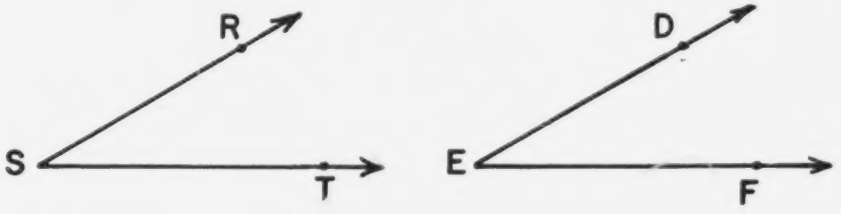
POLYGONS THAT ARE THE SAME SHAPE AND SIZE ARE CONGRUENT.

Triangle ABC is congruent to triangle XYZ ($\triangle ABC \cong \triangle XYZ$).



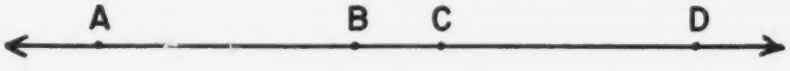
ANGLES WHICH CAN BE MADE TO COINCIDE ARE CONGRUENT.

Angle RST is congruent to angle DEF ($\angle RST \cong \angle DEF$).

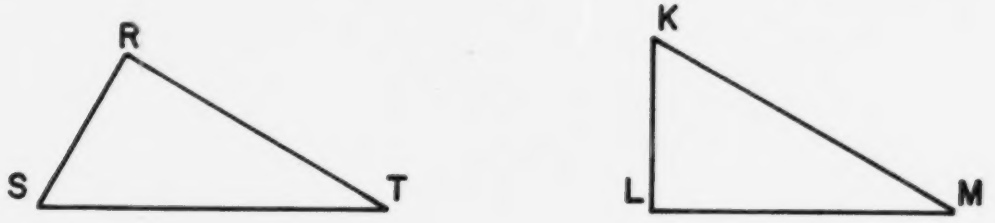


LINE SEGMENTS WHOSE END POINTS CAN BE MADE TO COINCIDE ARE CONGRUENT.

Segment AB is congruent to segment CD ($\overline{AB} \cong \overline{CD}$).



Triangle RST is congruent to triangle LKM ($\triangle RST \cong \triangle LKM$).



When we say triangle RST is congruent to triangle LKM or write $\triangle RST \cong \triangle LKM$, we understand this to mean that:

1. triangle RST corresponds to triangle LKM ($\triangle RST \leftrightarrow \triangle LKM$),
2. the corresponding angles are congruent.
3. the corresponding sides are congruent.

Since triangle RST is congruent to triangle LKM, the corresponding angles and sides are congruent.

$\angle R \cong \angle L$ $\quad\quad$ $\overline{RS} \cong \overline{LK}$

$\angle S \cong \angle K$ $\quad\quad$ $\overline{ST} \cong \overline{KM}$

$\angle T \cong \angle M$ $\quad\quad$ $\overline{TR} \cong \overline{ML}$

If you have forgotten how corresponding angles and sides are represented in a correspondence between two figures, turn back to pages 13 and 17.

DISCUSSION QUESTIONS

1. Compare the statement above concerning what is meant when we say two figures are congruent with the statement at the top of page 14. What is the same? What is different?
2. Are all congruent figures similar? Are all similar figures congruent?
3. $\triangle RST \leftrightarrow \triangle KLM$ does not show that the triangles above are congruent. Can you explain why?

EXERCISES

1. Name the congruent angles and sides of the congruent triangles named below.

   a. $\triangle HKJ \cong \triangle MNP$

   b. $\triangle BDF \cong \triangle XTZ$

   c. $\triangle RTV \cong \triangle LMN$

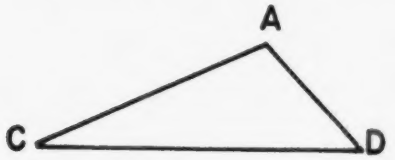
2. The congruent sides are given. Name the congruent figures.

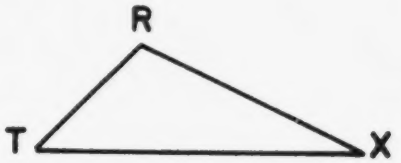
   a. $\overline{WX} \cong \overline{TU}$
   $\overline{XY} \cong \overline{UV}$
   $\overline{YW} \cong \overline{VT}$

   b. $\overline{BC} \cong \overline{HI}$
   $\overline{CD} \cong \overline{IJ}$
   $\overline{DA} \cong \overline{JK}$
   $\overline{AB} \cong \overline{KH}$

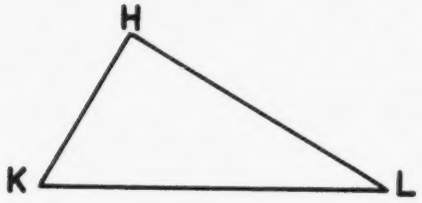
3. Name the pairs of congruent triangles drawn below.

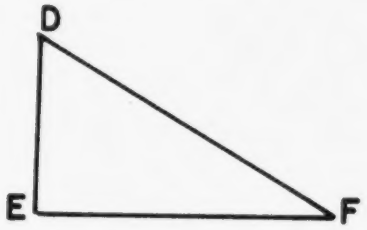
   a. $\triangle ACD \cong$ _____



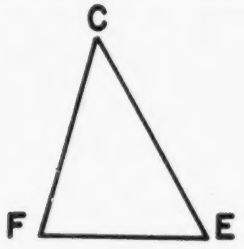


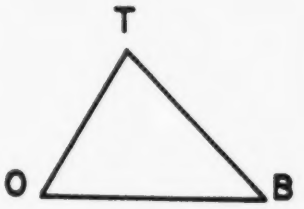
   b. $\triangle HKL \cong$ _____





   c. $\triangle TOB \cong$ _____

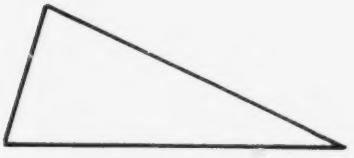




SIMILAR OR CONGRUENT?

Study the pairs of figures. Indicate whether you think they are similar, similar and congruent, or not similar by underlining the proper word or words.

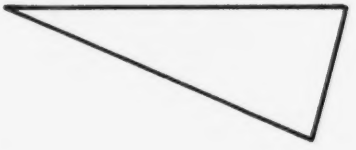
If you cannot tell just by looking, try tracing one of the figures on a piece of scrap paper and laying it on top of the other.

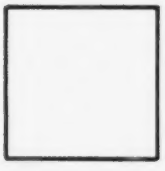


similar

congruent

not similar

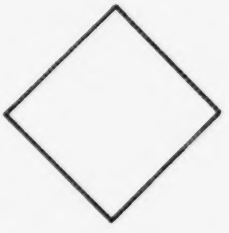


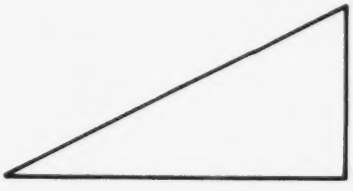


similar

congruent

not similar

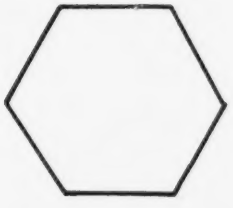




similar

congruent

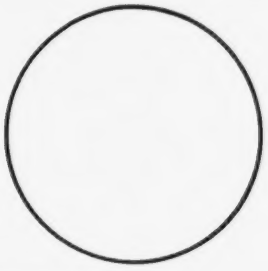
not similar



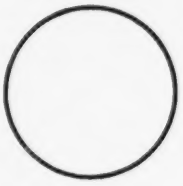
similar

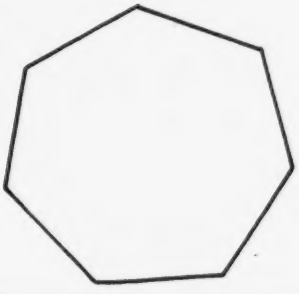
congruent

not similar

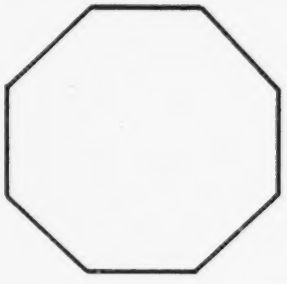


similar
congruent
not similar



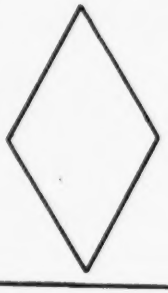


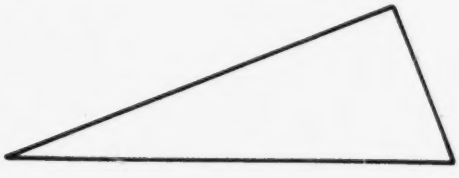
similar
congruent
not similar



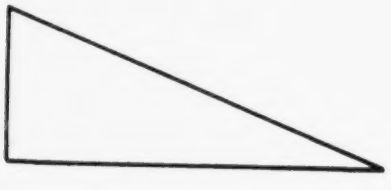
similar
congruent
not similar

similar
congruent
not similar

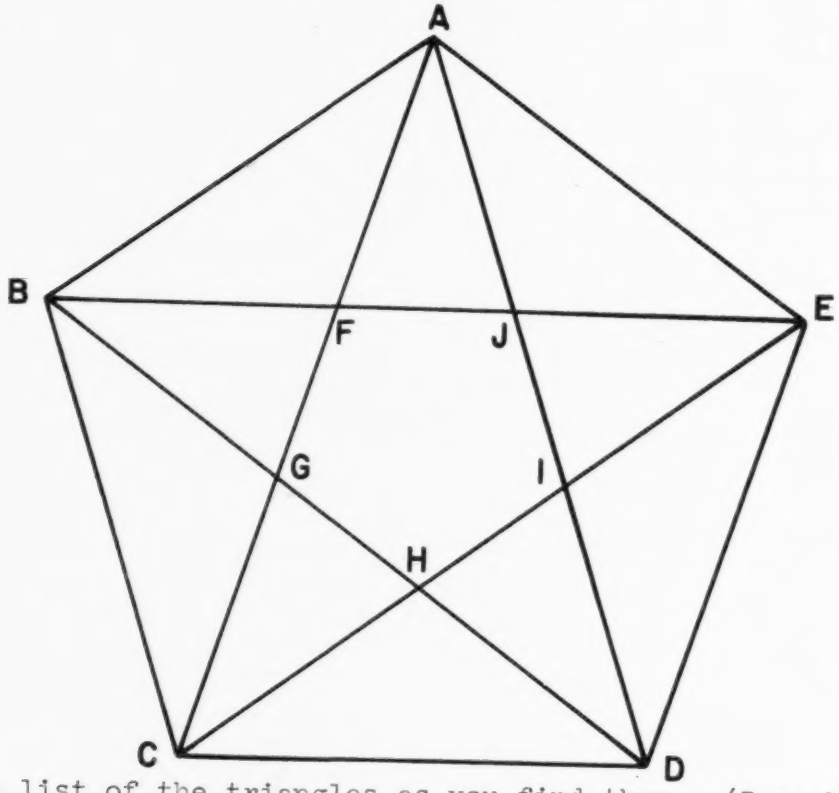




similar
congruent
not similar



HOW MANY TRIANGLES CAN YOU FIND?



Make a list of the triangles as you find them. (Remember: △ABC and △CBA are the same triangle.)

1. Name some triangles that are congruent to △ABC._____

2. Name some triangles that are congruent to △AFE._____

3. Name some triangles that are congruent to △FCE._____

4. Name some triangles that are similar but not congruent to △ACD._____

5. Name some triangles that are similar but not congruent to △ABE._____

6. Name a figure that is similar to figure ABCDE._____
   What is the name for a five-sided figure?_____

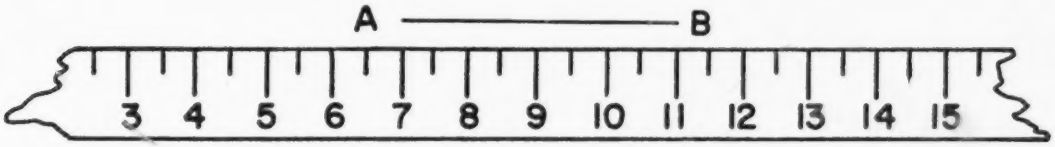
7. Make a small sketch of a figure similar to figure AGDE.

8. Name some figures that are congruent to figure AGDE._____

9. Make a small sketch of a figure similar to figure BGCE.

10. Make a small sketch of a figure similar to figure BCDE.

MEASURE AND RATIO



The measure, to the nearest centimeter, of segment AB is four centimeters. We write: $m(\overline{AB}) = 4cm$. Measure each of the following segments to the nearest centimeter and write the ratio of their measure to the measure of $\overline{AB}$.

C ——————————— D

$m(\overline{CD})$ = _____, Ratio: $\frac{m(\overline{CD})}{m(\overline{AB})}$ =

X ——— Y

$m(\overline{XY})$ = _____, Ratio: $\frac{m(\overline{XY})}{m(\overline{AB})}$ =

R ——————————————————— S

$m(\overline{RS})$ = _____, Ratio: $\frac{m(\overline{RS})}{m(\overline{AB})}$ =

K ——————————— L

$m(\overline{KL})$ = _____, Ratio: $\frac{m(\overline{KL})}{m(\overline{AB})}$ =

M ——————————————————— N

$m(\overline{MN})$ = _____, Ratio: $\frac{m(\overline{MN})}{m(\overline{AB})}$ =

U _____ V

$m(\overline{UV})$ = _____, Ratio: $\frac{m(\overline{UV})}{m(\overline{AB})}$ =

T _____ F

$m(\overline{TF})$ = _____, Ratio: $\frac{m(\overline{TF})}{m(\overline{AB})}$ =

G _____ H

$m(\overline{GH})$ = _____, Ratio: $\frac{m(\overline{GH})}{m(\overline{AB})}$ =

EXERCISES

1. From the segments above and on page 32, name the pair whose measures are in the ratio $\frac{5}{8}$. _____

2. The ratio of the measures of $\overline{XY}$ to $\overline{AB}$ is $\frac{2}{4}$. Is the ratio equal to the ratio $\frac{1}{2}$? Name at least three other pairs of segments whose measures are in the ratio $\frac{1}{2}$.

_____

3. The ratio of the measures of $\overline{AB}$ to $\overline{RS}$ is $\frac{1}{3}$. This is recorded in the chart below. Complete the chart by placing an "X" in the proper space of each column.

	$\frac{m(\overline{AB})}{m(\overline{RS})}$	$\frac{m(\overline{UV})}{m(\overline{KL})}$	$\frac{m(\overline{TF})}{m(\overline{RS})}$	$\frac{m(\overline{GH})}{m(\overline{RS})}$	$\frac{m(\overline{GH})}{m(\overline{TF})}$	$\frac{m(\overline{GH})}{m(\overline{AB})}$	$\frac{m(\overline{XY})}{m(\overline{KL})}$	$\frac{m(\overline{MN})}{m(\overline{RS})}$	$\frac{m(\overline{KL})}{m(\overline{CD})}$	$\frac{m(\overline{XY})}{m(\overline{CD})}$
$\frac{1}{3}$	X									
$\frac{1}{4}$										
$\frac{5}{6}$										
$\frac{3}{4}$										

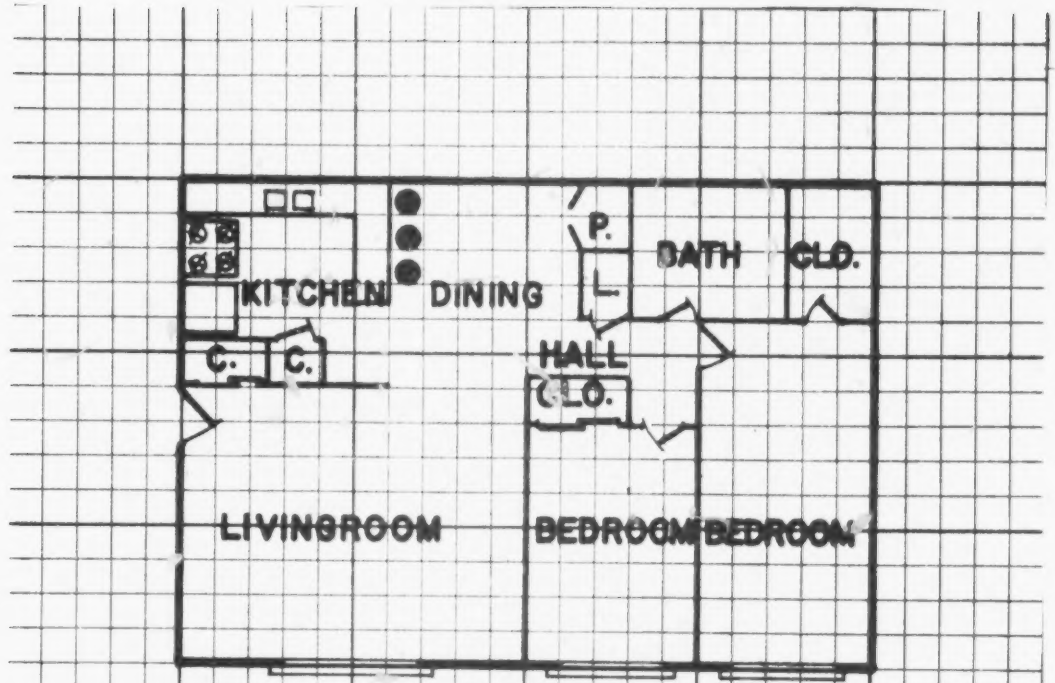


FIGURE 1

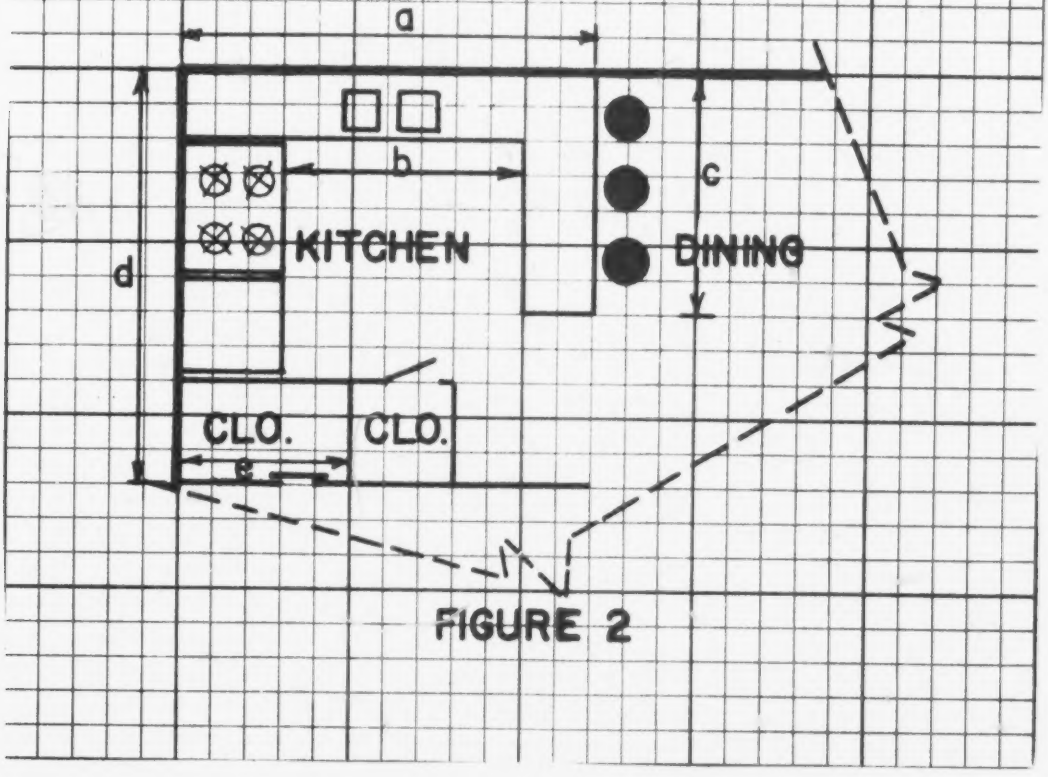


FIGURE 2

The diagram in figure 1 at the top of page 34 represents a floor plan for an apartment and figure 2 is an enlargement of the kitchen area. Using the side of one of the squares on the grid as a unit of length, find the following:

1. the lengths represented by a, b, c, d, and e in the enlargement (figure 2).
2. the lengths of the corresponding parts in the total floor plan (figure 1).
3. the ratios of the lengths in 1 to the lengths in 2.

Record the information in the table below.

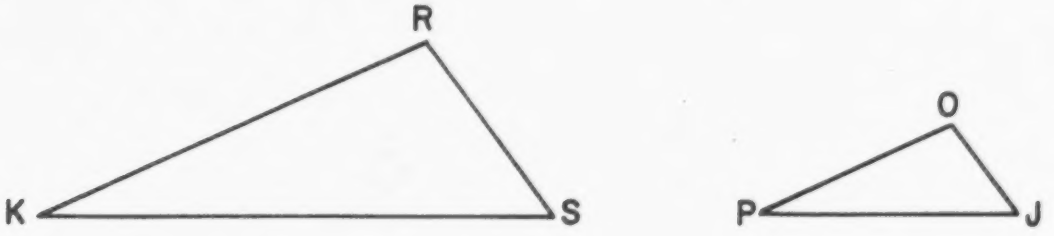
	ENLARGEMENT	TOTAL PLAN	RATIO
a			
b			
c			
d			
e			

DISCUSSION QUESTIONS

1. An architect would say the ratio of the enlargement to the total floor plan is $\frac{2}{1}$. Can you explain why?
2. Are the drawings of the kitchen areas congruent? Are they similar? Explain your answers.
3. The dimensions of the living room in fig. 1 are 8 by 10. What would the dimensions be if an enlargement were made in the ratio $\frac{2}{1}$?
4. Using the ratio $\frac{2}{1}$ to enlarge the total floor plan, what would be the new dimensions of the bathroom, small bedroom, large bedroom and walk-in closet?
5. If each unit of length in fig. 1 represented two feet, what would be the dimensions of the actual living room? The other rooms?

In each exercise, the figures are similar. Cut out the ruler at the bottom of this page and measure the sides of the figures to the nearest unit. Name a similarity correspondence and compare the ratios of the corresponding sides.

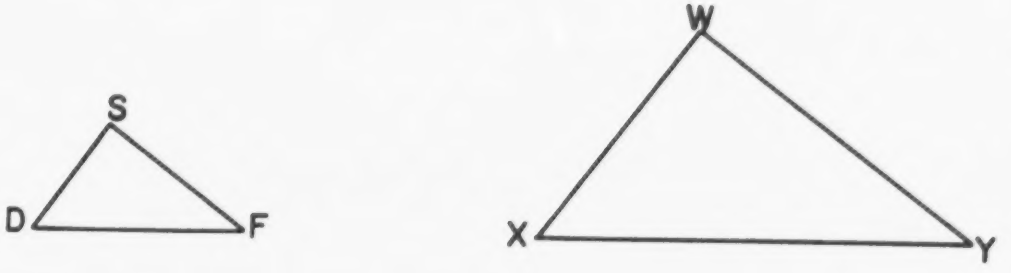
1.



$\triangle RKS \sim$ _____

$\frac{m(\overline{RK})}{m(\overline{OP})} =$ $\frac{m(\overline{KS})}{m(\overline{PJ})} =$ $\frac{m(\overline{SR})}{m(\overline{JO})} =$

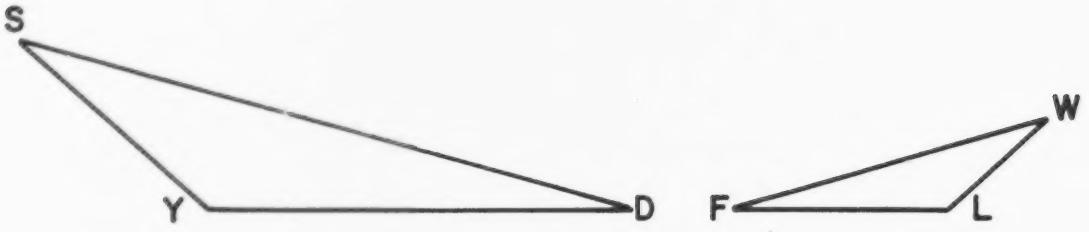
2.



$\triangle DFS \sim$ _____

$\frac{m(\overline{DF})}{m(\overline{XY})} =$ $\frac{m(\overline{FS})}{m(\overline{YW})} =$ $\frac{m(\overline{SD})}{m(\overline{WX})} =$

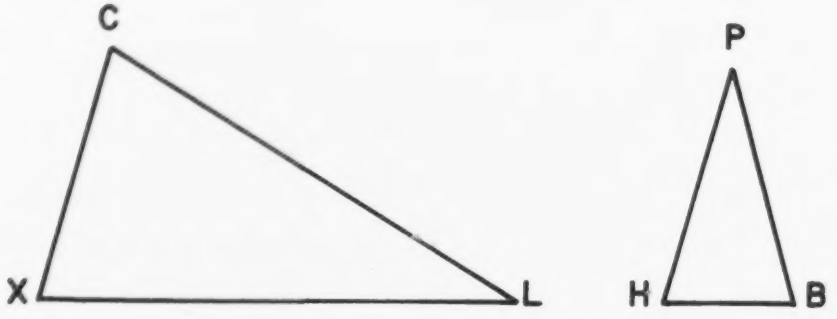
3.



Δ SYD $\sim$ _____

$\frac{m(\overline{SY})}{m(\overline{WL})} =$ $\frac{m(\overline{YD})}{m(\overline{FL})} =$ $\frac{m(\overline{DS})}{m(\overline{FW})} =$

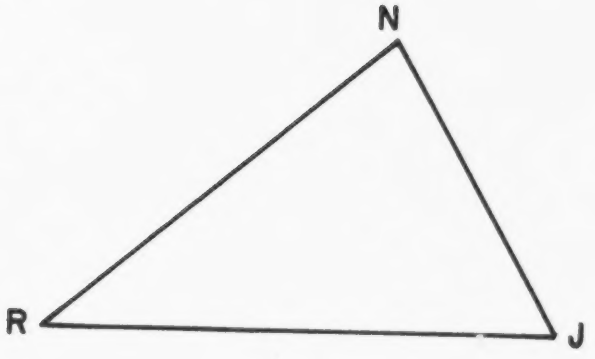
4.

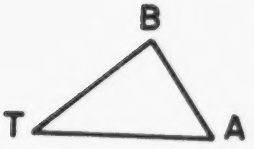


Δ XLC $\sim$ _____

$\frac{m(\overline{XL})}{m(\overline{BP})} =$ $\frac{m(\overline{LC})}{m(\overline{PH})} =$ $\frac{m(\overline{CX})}{m(\overline{HB})} =$

5.





$\triangle$ NRJ $\sim$ _____

$\frac{m(\overline{NR})}{m(\ \ )} =$ $\frac{m(\overline{RJ})}{m(\ \ )} =$ $\frac{m(\overline{JN})}{m(\ \ )} =$

6.

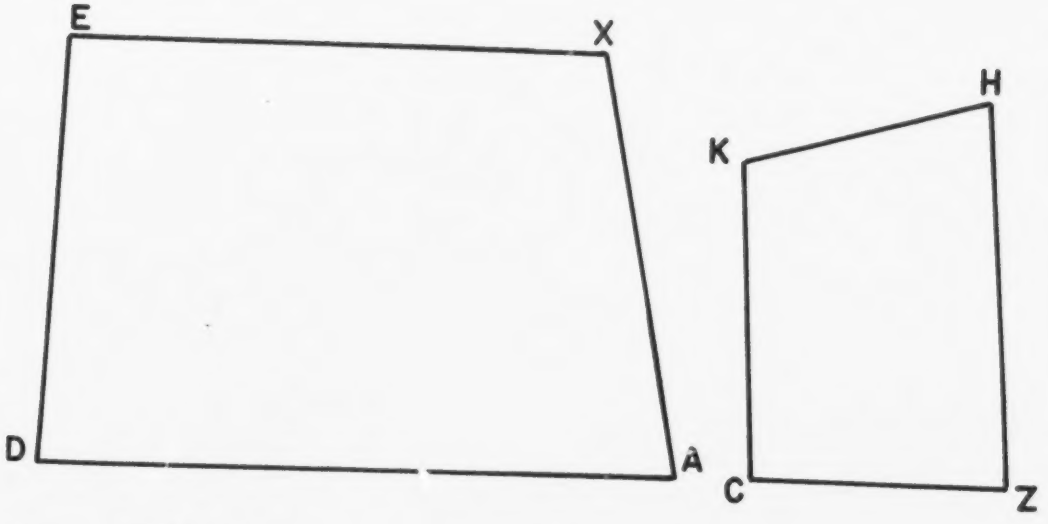


fig. EDAX $\sim$ _____

$\frac{m(\overline{ED})}{m(\ \ )} =$ $\frac{m(\overline{DA})}{m(\ \ )} =$ $\frac{m(\overline{AX})}{m(\ \ )} =$ $\frac{m(\overline{XE})}{m(\ \ )} =$

DISCUSSION QUESTIONS

1. In exercises 1 - 6, you were asked to compare the ratios of the corresponding sides of the similar figures. Discuss the results of your comparisons. Give examples.

2. How can you tell if two ratios are equal?

3. The lengths of segments a, b, c and d are:

$a = 6$ units
$b = 24$ units
$c = 8$ units
$d = 2$ units

Which of the following ratios are equal?

a) $\frac{a}{b}$ and $\frac{d}{c}$

b) $\frac{c}{b}$ and $\frac{d}{a}$

c) $\frac{a}{d}$ and $\frac{c}{b}$

d) $\frac{a}{b}$ and $\frac{c}{d}$

e) $\frac{b}{c}$ and $\frac{a}{d}$

f) $\frac{d}{b}$ and $\frac{a}{c}$

CLASS ACTIVITY

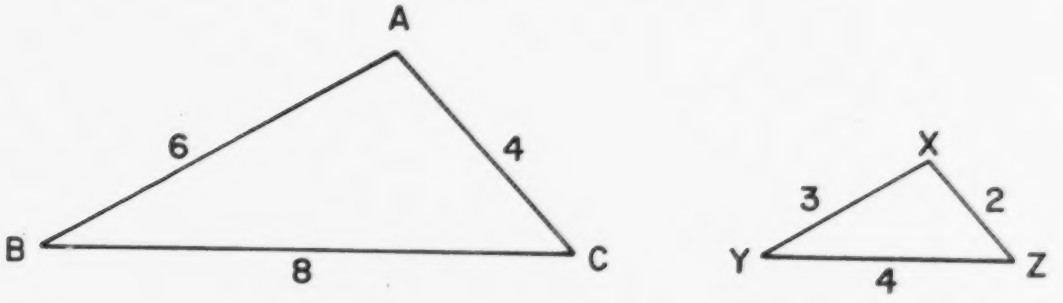
Use the grid paper at the back of this booklet to make a diagram of the entire floor plan on page 34, in the ratio of $\frac{2}{1}$.

EQUAL RATIOS

The exercises in Lesson 7 are an illustration of the relationship between the corresponding sides of similar figures. This relationship can be stated:

THE CORRESPONDING SIDES OF SIMILAR FIGURES HAVE EQUAL RATIOS.

The measures (to the nearest centimeter) are given for each of the sides of the similar triangles below.



Compare the ratios of the corresponding sides.

$\overline{AB} \longleftrightarrow \overline{XY}$ RATIO $\frac{m(\overline{AB})}{m(\overline{XY})} = \frac{6}{3}$

$\overline{BC} \longleftrightarrow \overline{YZ}$ RATIO $\frac{m(\overline{BC})}{m(\overline{YZ})} = \frac{8}{4}$

$\overline{CA} \longleftrightarrow \overline{ZX}$ RATIO $\frac{m(\overline{CA})}{m(\overline{ZX})} = \frac{4}{2}$

Is $\frac{6}{3}$ equal to $\frac{8}{4}$?

Is $\frac{8}{4}$ equal to $\frac{4}{2}$?

Is $\frac{4}{2}$ equal to $\frac{6}{3}$?

Is $\frac{m(\overline{AB})}{m(\overline{XY})} = \frac{m(\overline{BC})}{m(\overline{YZ})}$?

Is $\frac{m(\overline{BC})}{m(\overline{YZ})} = \frac{m(\overline{CA})}{m(\overline{ZX})}$?

Is $\frac{m(\overline{CA})}{m(\overline{ZX})} = \frac{m(\overline{AB})}{m(\overline{XY})}$?

Is $\frac{m(\overline{AB})}{m(\overline{XY})} = \frac{m(\overline{BC})}{m(\overline{YZ})} = \frac{m(\overline{CA})}{m(\overline{ZX})}$?

Notice that $\frac{m(\overline{BC})}{m(\overline{XY})}$ is <u>not</u> equal to $\frac{m(\overline{YZ})}{m(\overline{BC})}$. Can you explain why?

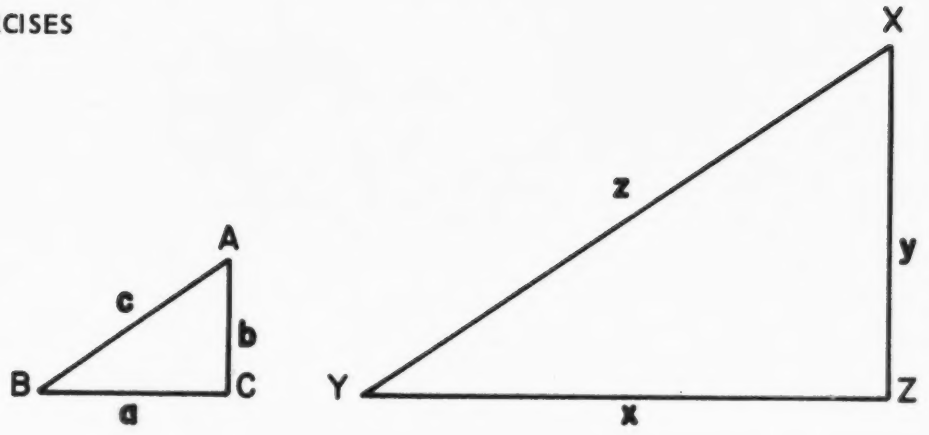
HINT! Why isn't $\frac{m(\overline{AB})}{m(\overline{XY})}$ equal to $\frac{m(\overline{XY})}{m(\overline{AB})}$?

When comparing ratios of corresponding sides of similar figures A and B, if the first ratio compares a side of figure A to a side of figure B, each other ratio must compare a side of figure A to a side of figure B.

If the first ratio compares a side of figure B to a side of figure A, what must the other ratios compare?

EXERCISES

1.



The sides of triangle ABC have lengths a, b and c; and the sides of triangle XYZ have lengths x, y and z. If the triangles are similar, which of the following statements are true?

a) $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

b) $\frac{a}{z} = \frac{b}{y} = \frac{c}{x}$

c) $\frac{y}{b} = \frac{x}{a} = \frac{z}{c}$

d) $\frac{x}{b} = \frac{y}{a}$

e) $\frac{y}{b} = \frac{x}{a}$

f) $\frac{c}{z} = \frac{b}{y}$

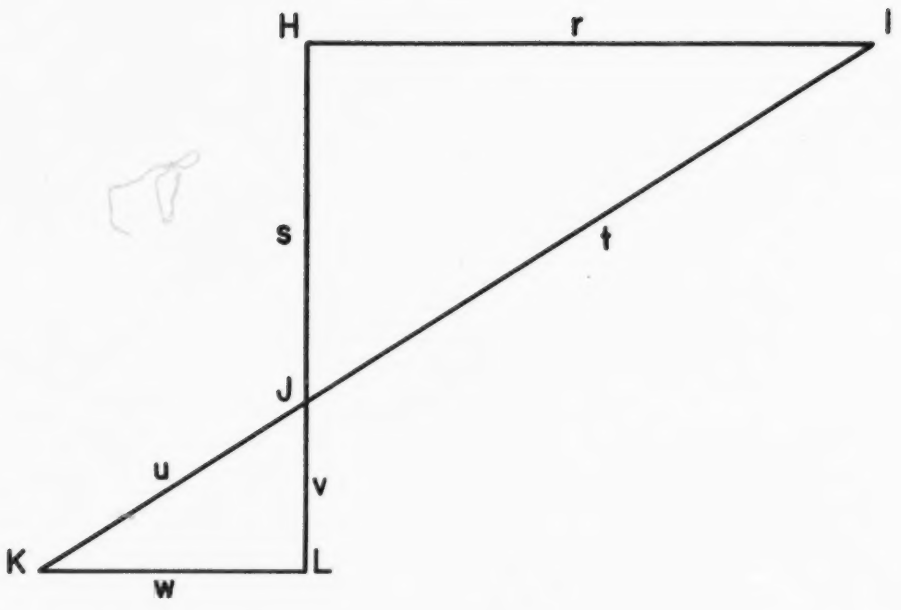
If b = 5 and y = 12, then $\frac{b}{y} = \frac{5}{12}$. Name the other two ratios which would be equal to $\frac{5}{12}$.

_____

If b = 5 and y = 12, name the ratios which would be equal to $\frac{12}{5}$.

_____

2.



The sides of triangle HIJ have lengths r, s and t; and the sides of triangle JKL have lengths u, v and w. The triangles are similar.

Name a similarity correspondence._____

Name the corresponding sides. _____

Which of the following statements are true?

a) $\frac{r}{u} = \frac{s}{v} = \frac{t}{w}$

b) $\frac{r}{w} = \frac{s}{v} = \frac{t}{u}$

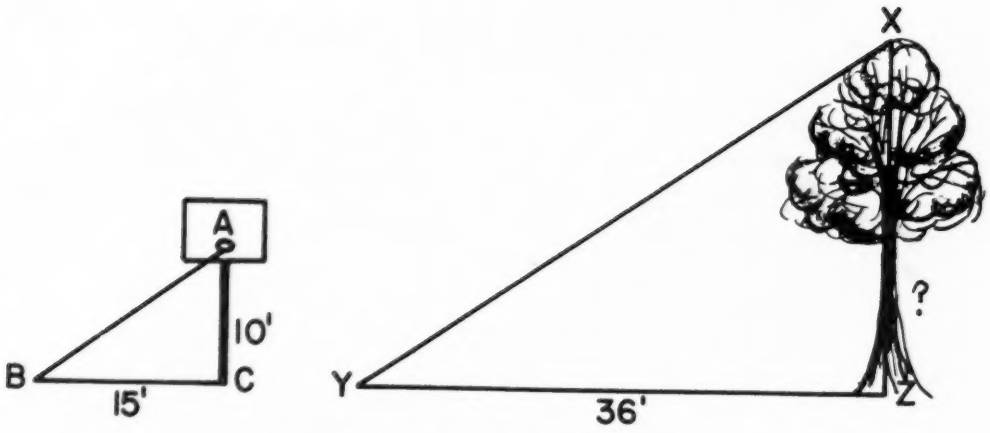
c) $\frac{u}{r} = \frac{v}{t} = \frac{w}{s}$

d) $\frac{r}{w} = \frac{t}{u}$

e) $\frac{r}{u} = \frac{u}{t}$

f) $\frac{s}{v} = \frac{r}{w}$

3.



A tree casts a shadow of 36 feet at a time when a nearby basketball hoop's shadow is 15 feet from the base of the supporting pole. By using the ratios of the measures of corresponding sides of similar triangles, it is possible to find the height of the tree.

In the illustration of the problem, two similar triangles have been drawn. Compare the triangles with those in exercise 1 on page 42.

Segment _____ represents the height of the tree.

Segment _____ represents the height of the basketball hoop.

Segment _____ represents the length of the tree's shadow.

Segment _____ represents the length of the hoop's shadow.

What does $\overline{AB}$ represent? _____

_____

What does $\overline{XY}$ represent? _____

_____

Why is the measure of $\overline{AC}$ equal to 10 feet? _____

_____

Place an "h" beside the question mark to represent the height of the tree.

Name the corresponding sides of the triangles.

Since the triangles are similar, the following statement is true. Why?

$$\frac{m(\overline{BC})}{m(\overline{YZ})} = \frac{m(\overline{AC})}{m(\overline{XZ})}$$

Substitute and solve (can you explain each step?).

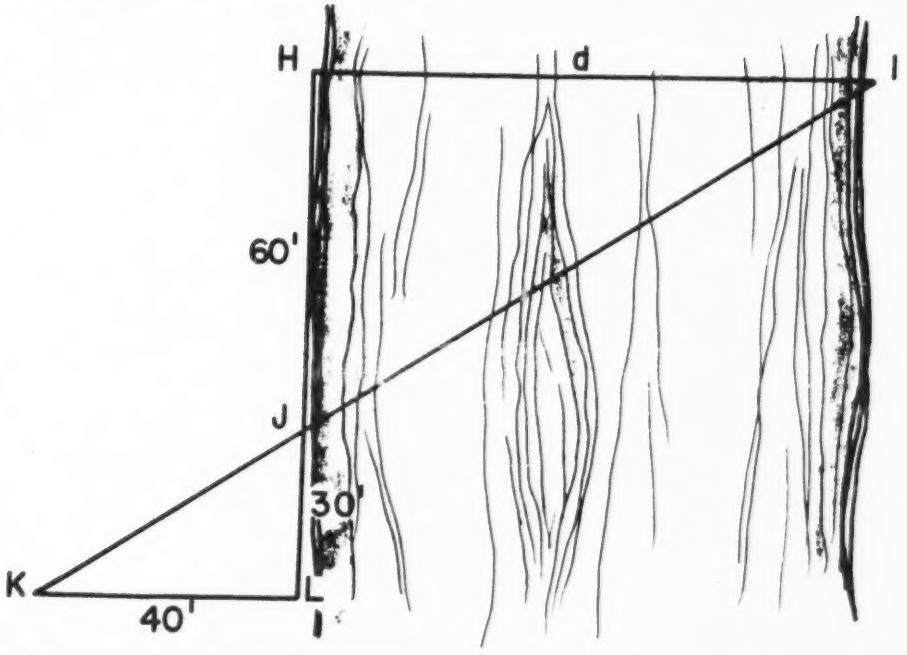
$$\frac{15}{36} = \frac{10}{h}$$

$$15h = 360$$

$$h = 24'$$

DISCUSSION QUESTIONS

1.



By using surveying instruments and the ratios of the corresponding sides of similar triangles, a team of civil engineers was able to determine the length of a bridge required to cross a river.

In the illustration of the problem above, triangle HIJ is similar to triangle LKJ (compare the illustration with the triangles in problem 2 on page 43).

Since the triangles are similar, the following statement is true. Why?

$$\frac{m(\overline{HI})}{m(\overline{LK})} = \frac{m(\overline{HJ})}{m(\overline{LJ})}$$

Substitute and solve (can you explain each step)?

$$\frac{d}{40} = \frac{60}{30}$$

$$30d = 2400$$

$$d = 80'$$

2.

The length of the Mustang pictured above is $5\frac{3}{4}$ inches. Ford Motor Company reports that the length of an actual Mustang is 187 inches. The ratio of the length of the car in the picture of the length of an actual car is $\frac{5\frac{3}{4}}{187}$.

$\frac{5\frac{3}{4}}{187}$ is approximately equal to $\frac{1}{32}$.

We write: $\frac{5\frac{3}{4}}{187} \approx \frac{1}{32}$.

a. If the ratio of the length of the picture to the actual car is $\frac{1}{32}$, would the ratio of the heights also be $\frac{1}{32}$? Would the ratio of the lengths of the doors be $\frac{1}{32}$?

   Name several other items whose ratios would be $\frac{1}{32}$.

b. Would the ratio of the length of the car in the picture to the height of an actual car be $\frac{1}{32}$? Why?

c. "The corresponding sides of similar figures have equal ratios". Explain how this statement could be used to verify the correct answers to questions a and b.